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A PLANNING MODEL FOR THE OPTIMUM UTILIZATION OF MANPOWER RESOURCES

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A PLANNING MODEL FOR THE OPTIMUM UTILIZATION OF MANPOWER RESOURCES

by

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ABSTRACT

This thesis demonstrates the feasibility of constructing a network flow model to represent the U.S. Navy officer personnel system. This model consists of nodes connected by directed arcs which represent, respectively, career states and paths between these states. Flows moving over these arcs represent the movements of officers from state to state through time. A measure is developed which relates planning effectiveness to the dollar costs incurred by the Navy in recruiting, training, and maintaining officers. The network flow model is then equated to a linear program which can be solved for the dynamic flows of officers necessary to meet expected future requirements with maximum planning effectiveness. An example problem is hypothesized and solved to illustrate the technique. The author recommends that a small scale operational model be constructed to represent a segment of the Navy Officer Corps in order to better estimate the value of this approach to officer personnel planning in the Navy.

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1. Introduction.

Personnel management may be divided into two broad areas which can be simply described as follows:

(1) How to provide the personnel resources required?

(2) How to distribute those resources to meet requirements?

A third area, the determination of personnel requirements, is closely related to, but is not considered to be a part of, the personnel management problem.

For convenience, the first area will be called the personnel planning problem and the second will be referred to as the personnel distribution problem. The two areas are not mutually exclusive because good planning must include cognizance of distribution policies and good planning will facilitate distribution; however, it is possible to consider each area as a separate problem if these relationships are considered. We will consider the planning problem herein.

The personnel planning problem can be defined as follows: How can personnel planners best meet expected future requirements for personnel with available resources and within existing legal and administrative constraints? This thesis proposes that a network flow model can be developed which will assist personnel managers in solving the personnel planning problem associated with the U.S. Naval Officer Corps.

Arguments will proceed along the following general lines: First, the nature and general characteristics of the officer personnel system will be discussed. Then we will show how that system can be represented by a model consisting of nodes and arcs, with flows moving between nodes of the system. The next step will be to present a method for solving such a network and to show why a measure of planning effectiveness is required. This measure will then be derived. At this point, the model will be developed and an example problem will be stated and solved to illustrate how the ideas contained herein can be applied. Next, the model

will be discussed in terms of assumptions, limitations, advantages and disadvantages of the technique; and, finally, computational difficulties will be considered.

2. Theory of the Model.

This section describes the officer personnel system and shows how such systems may be generally represented by network flow models.

2.1. Characteristics of the Officer Personnel System. The U.S. Navy officer personnel system can be described as both large and complex. It consists of more than 76,000 officers of diverse qualification, rank, and experience level.(1) Primary qualifications are reflected by one of several designator codes, such as 11XX, 13XX, and 31XX, which represent line officers, line officers with aviation qualification, and Supply Corps officers, respectively. Secondary qualifications are reflected by sub-specialty codes representing advanced training or significant specialized experience in personnel administration, aeronautical engineering, operations analysis, etc. Further qualification levels are used as necessary to identify technical competence in particular types of ships, submarines or aircraft. There are 10 different ranks of commissioned officers with experience levels ranging from 0 to more than 40 years. The system is large and complex because requirements for personnel are large and diverse. The requirements are also unique so that the Navy must, in general, train most of its officers "in house".

Second, the system is constrained; i.e., there are several legal and administrative constraints on the numbers and types of officers allowed. Additionally, there are administrative policies to observe which act as constraints. Examples of legal constraints are Title 10 U.S. Code and the Stennis Ceiling. Title 10 U.S. Code specifies the grade distribution, minimum times in grade, minimum service requirements for retirement and promotion, limited duty officer percentages allowed, etc. for naval officers. The so-called Stennis Ceiling is a

Congressional constraint which limits the number of flag officers (rear admiral and above) on active duty to not more than 302.[2] An example of an administrative constraint is the implementation of the "Kieth Board" recommendation to limit the number of engineering duty officers to 881.[3] An example of an administrative policy which acts as a constraint is the current policy of not accepting applications for augmentation into the Regular Navy from Naval Reserve aviators who have attained the permanent rank of lieutenant commander.[4]

In addition, the requirements for officers change frequently as the weapons systems and tasks of the Navy change. Obviously, a substantial amount of planning is required in order to have sufficient numbers of qualified personnel available to meet requirements.

2.2. The Network Flow Model. The model we propose to use is known as a network flow model. Such models constitute a sub-class of linear programming models and have been used previously by other authors to represent personnel systems. For instance, Merck and Ford so described a portion of the U.S. Air Force enlisted personnel system in 1959[5]; Gorham developed and solved a network flow model which represented an Air Force personnel training problem in 1960[6]; and Hayter and Conner used a similar approach on a problem involving the U.S. Navy enlisted personnel system in 1965.[7]

Every officer in the U.S. Navy can be described in terms of rank, designator, subspecialty, and current employment. For instance, we might accurately describe one individual as being an Ensign, 11XX, no subspecialty, serving as navigator on a destroyer. Another individual could be described as a Lieutenant, 11XX, personnel administration, serving as executive officer on a destroyer. Since the above descriptions could apply equally well to several officers, we have in effect defined subsets of the naval officer population. By proceeding to form all possible combinations of the four characteristics, we will exhaustively partition the naval officer population into a collection of mutually

exclusive subsets. We will call such subsets career states. (Note: career states could be formed with other characteristics if desired.) A similar partition of the naval officer population could be made next year or the year after, etc., but the officers who are in a particular career state today might not be in that same career state in subsequent time periods. In fact, officers routinely move from one state to another as time passes. If we represent each career state as a labeled node and the path of movement from one state to another by a directed arc from one node to another, we could draw a picture of the naval officer personnel system. By adding a source node from which personnel move into the Navy and a sink node to which personnel move upon leaving the Navy, we can complete the picture.

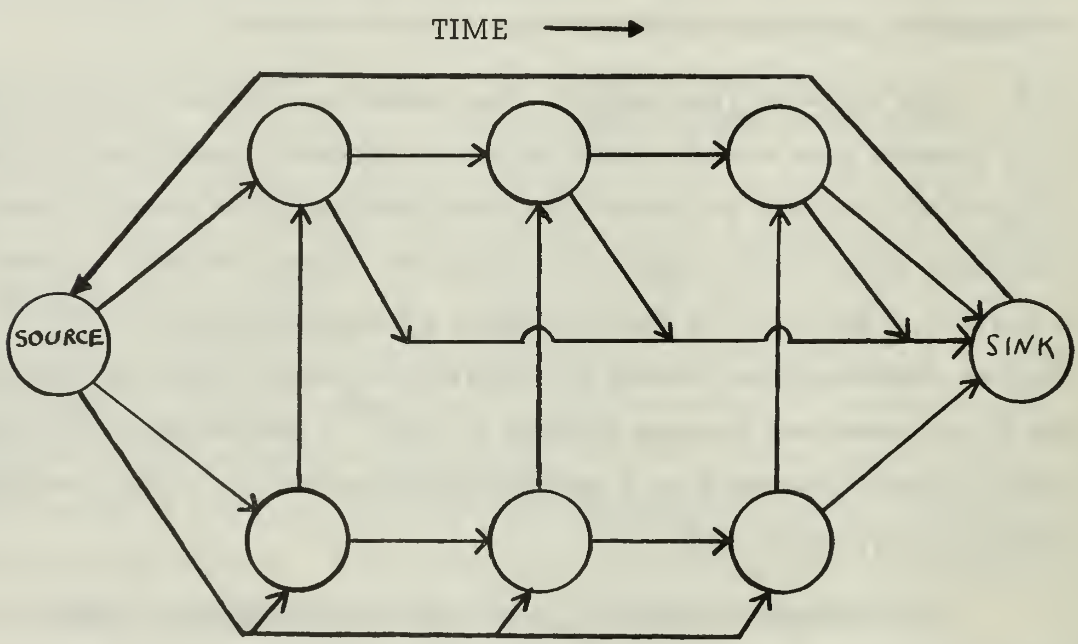


FIGURE 1

Figure 1 is a simplified example of a personnel system where the nodes are depicted as circles and the arcs between nodes are shown as straight lines. Note that arcs do not exist from each node to every other node. This is because many paths of movement are not possible in the real system.

The actual movement of personnel over an arc will be called a flow and denoted by f_{ij}, where f_{ij} represents the number of personnel moving from node i to node j.

With each arc we can associate three integers. These are called lower bound (L_{ij}), upper bound (M_{ij}), and cost (C_{ij}), where the subscripts denote the arc from node i to node j. In our network flow model we will use L_{ij} and M_{ij} to control the flow in arcs of the system, thereby imposing the personnel system constraints upon the network flow model. The C_{ij} will be defined in a manner that will allow us to measure planning effectiveness. The general approach will be to maximize planning effectiveness by minimizing these costs subject to the constraints of the model. Fulkerson has shown that networks of the foregoing type can be expressed as a linear program of the following form:[8]

$$\text{Minimize} \quad \sum_{ij} C_{ij} f_{ij}$$

$$\text{Subject to} \quad L_{ij} \le f_{ij} \le M_{ij} \qquad \text{for all arcs } ij$$

$$\text{and} \quad \sum_{j} (f_{ij} - f_{ji}) = 0 \qquad \text{for all nodes } i$$

Such linear programs can be efficiently solved by the Ford-Fulkerson Out-of-Kilter algorithm, which is given in Appendix I.

At this point we have described the naval officer personnel system and shown how it can be represented by a network flow model which can be solved for the set of flows which minimize costs subject to the given constraints. The set of flows thereby determined represents the solution to the personnel planning problem, for it is the plan which, if followed, will best meet the expected future requirements. The following sections will be devoted to showing how C_{ij}, L_{ij}, and M_{ij} are determined and incorporated into the model to accomplish the previously stated goals.

3. <u>Measure of Effectiveness</u>.

We need to define a cost which can be used in the model to measure our planning effectiveness. We will proceed on the hypothesis that if our plan programs a correctly qualified officer for each requirement, then our planning has been optimal. We will assume that this goal will not always be realized; therefore, we need to establish a method for evaluating the effectiveness of plans which do not program a correctly qualified officer into every billet. This shortcoming could occur in two ways:

(1) An officer is programmed for the requirement but is not correctly qualified for the job.

(2) An officer is not programmed for the requirement.

We will measure our planning effectiveness by assigning a dollar value to each planned assignment of officer to billet which is not optimum. This dollar value will be related to the magnitude of the departure from optimality. Then, by minimizing these values over the entire plan, we will maximize our planning effectiveness. These dollar values will be called penalty costs and form one part of the costs, C_{ij}, which will be used in the network flow model.

3.1. <u>Derivation of C_{ij}</u>. There are two basic types of costs associated with naval officers; these may be called investment costs and maintenance costs. Investment costs arise because of the fact that naval officers must be trained by the Navy and, in general, cannot be hired directly from the national labor force. Thus, the total costs incurred by the Navy in the course of recruiting, training, and maintaining an individual may be considered as an investment in the individual. As an example, consider Figure 2, below:

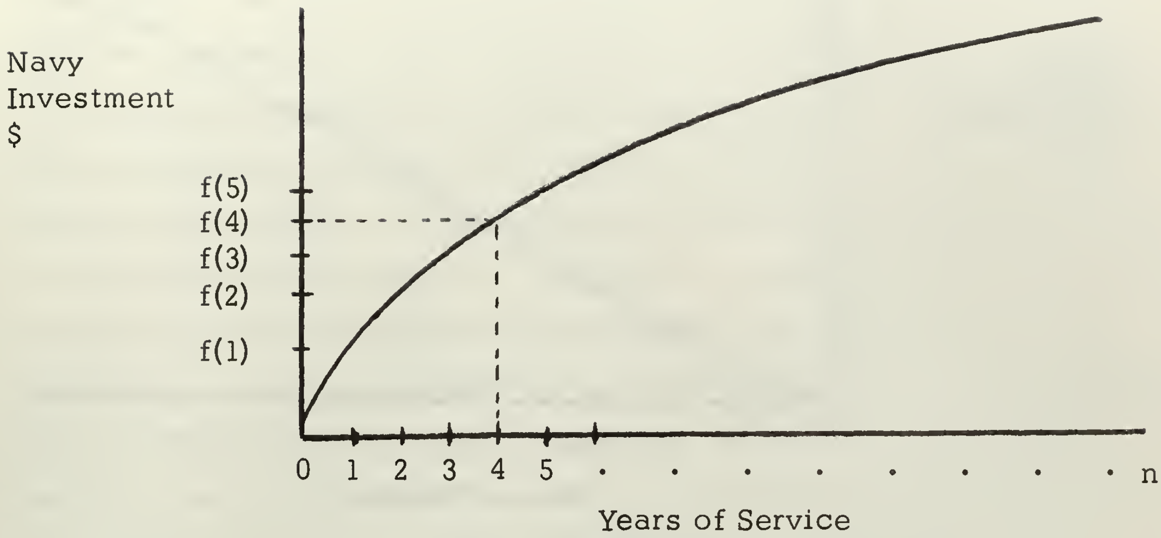


FIGURE 2**

(**Figures 2 and 3 are conveniently depicted as continuous curves when in reality they are step functions with the steps occurring at unequal time increments.)

Figure 2 could represent the total Navy investment in a hypothetical naval officer as a function of time. It should be noted that such a curve could be constructed for any naval officer, but not all curves would look exactly alike. All would be monotonically increasing and have the same general shape, but each curve would represent a particular career pattern. Even though summed maintenance costs are included in investment costs, it is useful to consider annual maintenance costs separately as illustrated in Figure 3, below:

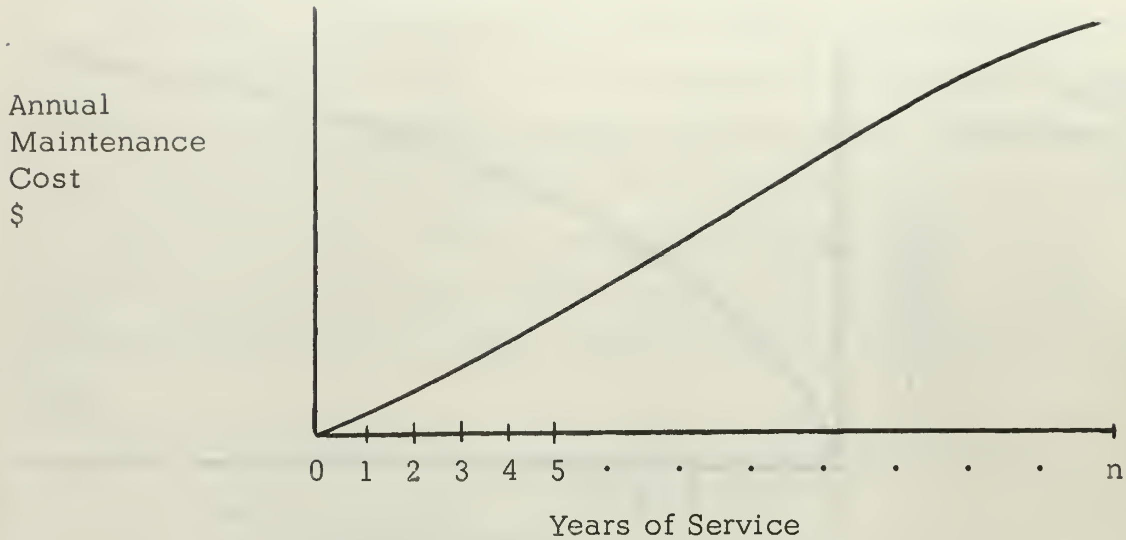


FIGURE 3

Annual maintenance costs are discussed in detail later in this section, but they may be broadly defined as all costs which regularly reoccur; for example, pay and allowances, transportation costs, Social Security contributions, etc. Again, Figure 3 is a fictitious graph showing the general shape of a curve which can be constructed for any naval officer. Such curves will not all be alike, but all will be monotonically nondecreasing.

Requirements are typically stated in terms of billets and a billet is essentially a description of a particular type of individual. It then follows that points on the abscissa of Figure 2 correspond to billets and imply that each billet has an investment cost associated with it. By similar reasoning, it can be argued that each billet also has a maintenance cost associated with it. Thus, requirements can be translated into costs. These costs are particularly important because they can be used to determine the relative worth of individuals to the Navy. The principle of revealed preference, as interpreted by Baumol, may be applied to support this contention. Briefly, the principle of revealed preference asserts that: "... if a consumer buys some collection of goods A, rather than any of the alternative collections B, C, D, etc.,

and it turns out that none of the latter is more expensive than A, we say that A has been 'revealed preferred' to the others ...." [9] Now assume that the Navy has a requirement for an officer whose qualifications exactly correspond to the point 4 on the abscissa of Figure 2. This implies that the Navy has "bought" or would "buy" this type of officer for that billet. Since officers with qualifications corresponding to points to the left of point 4 "cost less", we can assert by the principle of revealed preference that the officer with qualifications corresponding to point 4 is preferred to all points to the left of 4 <u>with regard to the particular billet we are discussing</u>. Since this is a planning problem, we could in theory increase the qualifications of officer 3 to the level of officer 4 by investing an amount of money in 3 equal to $f(4)-f(3)$; therefore, it must follow that 4 is preferred to 3 by an amount given by $f(4)-f(3)$. Similarly, 4 is preferred to 2 by an amount given by $f(4)-f(2)$, etc. This argument does not suffice for evaluating those officers whose qualifications are represented by points to the right of 4, but the extension is obvious. From a planning point of view, the assignment of an over-qualified officer to the billet we are discussing represents an excessive expenditure which is also a departure from optimality. The amount of this excess is given by $f(k)-f(4)$, where k is greater than 4.

By assumption, it will be necessary to assign officers to billets for which they are not exactly qualified. In practice, the possible range of non-optimal assignments for a particular type of officer may be quite broad. From a planning point of view, such assignments represent an undesirable departure from optimality. Quantitatively, this departure may be measured in dollars and regarded as a penalty cost. The magnitude of this penalty cost is the amount of money either positive or negative, which would be required to achieve optimality. This is illustrated in Figure 4, below:

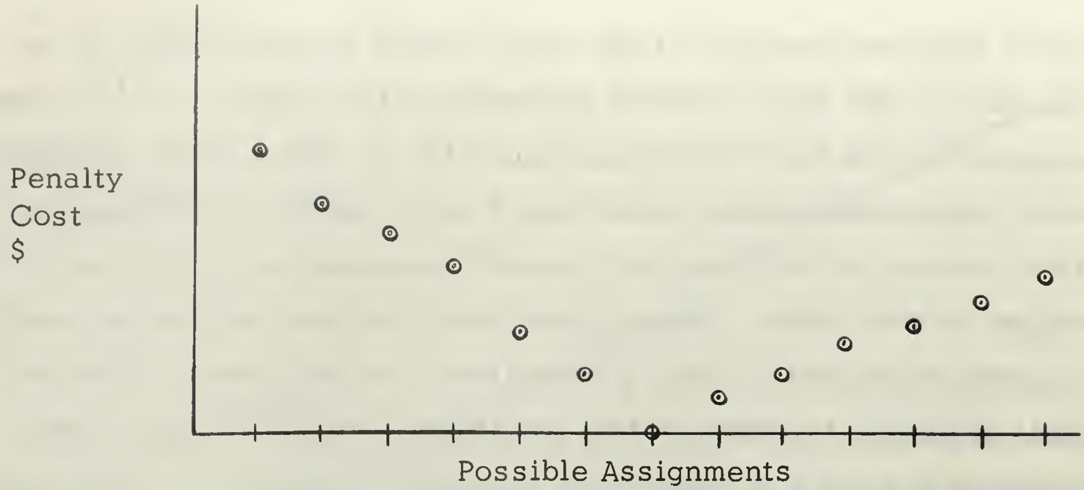


FIGURE 4*

(*Here we have also depicted a situation which is perhaps not typical, but it is true that the points on the abscissa could be so arranged to give a curve with this general shape. Many other curves are also possible, but all would have a minimum value.)

Figure 4 shows penalty costs as a function of assignment for a particular individual. Each type of officer would be expected to have a curve of similar but not identical shape using his possible assignments as the abscissa. Recalling from Figure 3 that each officer has a particular maintenance cost, Figure 5 can be constructed by adding this maintenance cost to Figure 4.

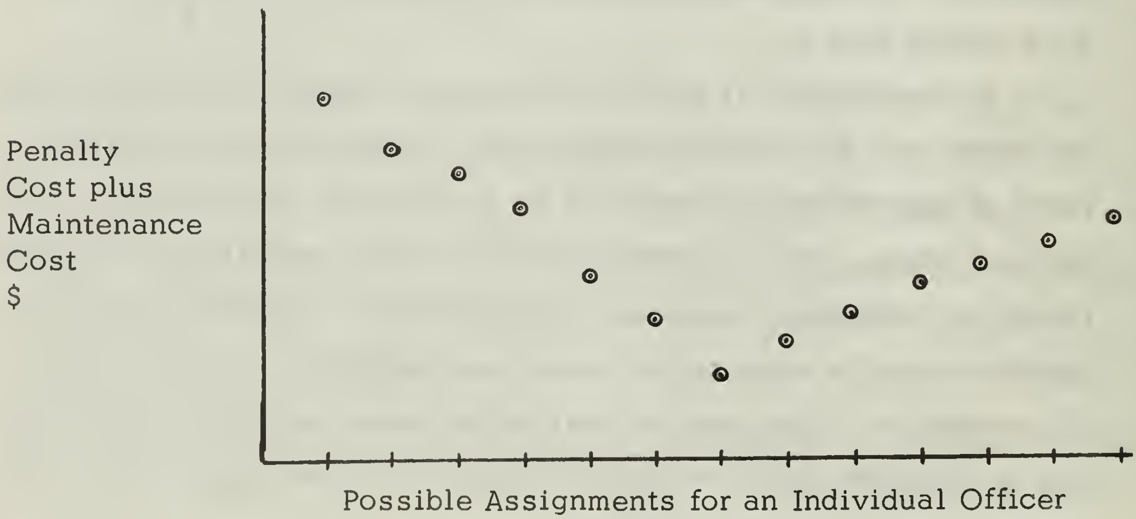


FIGURE 5

It is therefore possible to construct a particular Figure 5 for each officer in the Navy if we desired to do so. In general, we would expect to be able to assign one of several officers to a particular billet. In this event, it is then possible to construct Figure 6 which shows the penalty costs plus maintenance costs as a function of the various individuals who could be assigned to a particular billet.

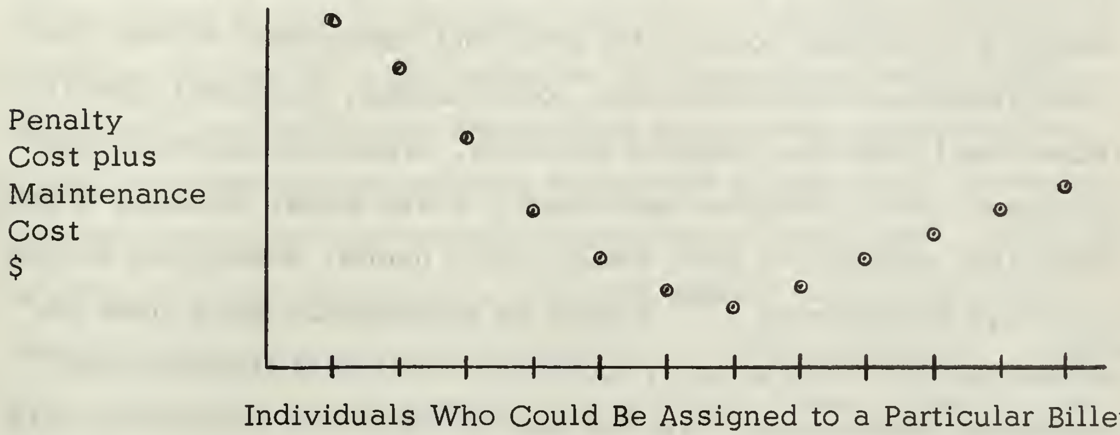


FIGURE 6*

(*Again, it would be necessary to "properly" arrange the points on the abscissa in order to come up with this shape curve.)

In particular, Figure 6 could be constructed for any billet in the Navy. The penalty cost plus maintenance cost associated with the assignment (movement from the i^{th} career state) to a particular billet (j^{th} career state) will be denoted by C_{ij}. Thus, we have derived the costs, C_{ij}, (for use in the Network Flow Model) from investment costs and maintenance costs. The next section will describe how these two costs may be estimated.

3.2. <u>Estimation of Investment Costs and Maintenance Costs</u>. Annual maintenance costs are defined to be the total cost to the Navy of maintaining an individual for one year where all personnel costs except training and recruiting are considered to be maintenance costs. Investment costs are then defined as the cumulative total costs incurred by the Navy to recruit, train, and maintain an individual to his current state of qualification.

Clearly the task of computing exact costs for each officer would be a monumental undertaking and so it seems desirable to estimate these costs. Maintenance costs will be considered first. The model works with career states; hence, we should estimate costs for these selected career states rather than individuals. This implies the existence of two types of maintenance costs; i.e., those that are a function of career states of interest and those that are not. Costs in the latter category include the overhead costs of the personnel management system, the costs associated with exchanges, commissaries, dependent schools, recreational facilities, medical facilities, government quarters, life insurance, dental facilities and others. A well known principle of cost estimating asserts that costs which have a neutral impact upon alternatives may be neglected.[10] Insofar as maintenance costs alone are considered, we could properly ignore the above cost elements, but cumulative maintenance costs are also an element of investment costs and, in general, would not have a neutral impact on the system alternatives. There is no easy way to handle costs of the type under discussion. It is not even clear that all should be assigned to the maintenance cost category. These difficulties force us to look at the other elements of maintenance costs (those that are a function of career state). These costs are:

(1) Pay and allowances

(2) Retirement allocation

(3) Transportation costs

(4) Unused leave pay

(5) Social Security contributions

Contrary to the situation prevailing with the first type of maintenance costs, the elements of this second group are easily estimated. Additionally, it has been stated that this second group includes the major portion of identifiable Navy personnel costs.[11] This latter assertion was additionally verified by consulting the Navy budget for

1966.[1] The costs are not identified in great detail, but it is possible to put an upper bound on the maintenance costs which are not a function of career state. Total military personnel costs budgeted as such are given as 3544 million dollars; 3541 million are for cost elements which are a function of career state which leaves 3 million for those costs which are not a function of career state.[1] The difficulty is that costs which are not a function of career state are not identified as personnel costs but are carried in the operation and maintenance funds (O&M). We cannot say how much of these funds are properly attributable to personnel costs but can certainly assert that the personnel portion is less than the total of all relevant categories of O&M funds. This total is 156 million for medical care, plus 395 million for service-wide administrative operations which sums to 551 million dollars.[1] Adding the 3 million previously identified as personnel costs, gives an upper bound of 554 million dollars for personnel maintenance costs which are not a function of career state. This means that if we neglect this category of costs and consider only those maintenance costs which are a function of career state, we will be underestimating our annual maintenance costs by at most 15%. For obvious reasons, plus the arguments previously made, this error is surely much less. In any event, the probable accuracy should be sufficient for use in the model.

Investment costs have been defined to consist of cumulative maintenance costs, recruiting costs, and training costs. Because we are costing career states rather than individuals, we cannot a priori stipulate the procurement source of the various career states. We therefore neglect recruitment costs since we could only include a common average figure whose inclusion would not influence the model outputs. Maintenance costs can be estimated as shown above and summed for inclusion as investment costs. This leaves only training costs to consider, and, according to Jackson,[11] cost per student estimates for most Navy training activities are available in one or the other of the following reports:

(1) "Costs for Non-Aviation and Postgraduate Type Schools"; Bureau of Naval Personnel, Pers C26; BuPers Report 1500-7.

(2) "Cost Per Student Report (Estimated Course Costs)"; Chief of Naval Air Technical Training, Code 341.

In summary then, it has been shown that the required cost inputs can be estimated from existing data on:

(1) Pay and allowances.

(2) Retirement allocation.

(3) Transportation costs.

(4) Unused leave pay.

(5) Social Security contributions.

(6) School/training costs.

These costs can then be transformed into penalty costs and then to penalty costs plus maintenance costs as a function of assignment by the arguments given in the previous section.

4. Formulation.

This chapter will discuss in detail how constraints and requirements can be accommodated in our network flow model. This discussion will be followed by sections dealing with gains, losses, and network synthesis, and, finally, an intuitive description of the total system model will be presented.

4.1. Constraints: Constraints are of two general types: flow constraints (limitations on movements between career states) and node constraints (limitations on the total numbers of particular types of officers allowed). Promotion from one rank to another is a classic example of the first type of constraint. A maximum promotion flow from node i to node j can be specified by setting the arc upper bound (M_{ij}) equal to the desired upper limit. A lower limit can also be specified by setting the arc lower bound (L_{ij}) equal to the minimum number of

promotions desired. All flow constraints can be incorporated in the model in this fashion. Node constraints are generally more difficult to model in that network modification is usually required.

Examples of the second type of constraint are the Title 10, U.S. Code restrictions on the numbers of naval officers who may be on active duty at any one time. These are as follows: Of the total number authorized by Congress, not more than 75/100 of 1% may be flag officers; not more than 6% may be captains; not more than 12% may be commanders; not more than 18% may be lieutenant commanders; and not more than 24-75/100% may be lieutenants. The remaining 38-1/2% to be ensigns and lieutenants (junior grade). The significant exception to the above is that carry-down is permitted; that is, if the 75/100 of 1% allowed for flag officers is not needed, the remaining allowance may be added to the percentage allowed in the captain rank; similarly for each of the other grades. Carry-down would be a difficult constraint to model, since we are trying to simultaneously model for more than one time period. Fortunately it can be treated as a given input with regard to the naval officer personnel planning problem because of the way it is currently used by the Navy. In general, carry-down is used to assist in solving higher level personnel problems such as retention and morale. For example, officer corps morale would probably be damaged if promotion opportunities were allowed to fluctuate significantly between adjacent year groups; therefore, carry-down can be used to dampen out the large fluctuations in promotion rates which could occur if we determined promotion rates only on the basis of immediate requirements. Another example is the current policy of "carrying down" part of the captain and commander Title 10 authorization to the lieutenant commander and lieutenant ranks in order to give young officers an opportunity for rapid promotion to those ranks. This rapid promotion is a career incentive for junior officers. We will therefore assume that a personnel planner would be given specific numerical constraints on the numbers of each rank. For convenience we will continue to refer to these given

numbers as Title 10 constraints. We can incorporate such node capacities by replacing each capacitated node with two nodes connected by a capacitated arc.[12] The arc will have its upper bound set to the constraint and the lower bound at zero. In most cases, node constraints will require such a network modification, although a few situations may arise wherein the constraint can be incorporated merely by adding an appropriate upper bound to an existing arc.

4.2. Requirements. We assume that the requirements for personnel are generated outside of the personnel management organization so that such requirements will be considered as a given input for our planning problem. We also assume that the Title 10 limitations are the binding constraints in the total planning problem. This assumption follows from the distribution policy of always assigning someone to a billet unless the limitations on total numbers of officers allowed prevent such assignment. Policy or administrative constraints would never be knowingly allowed to cause apparent manpower shortages. With these two assumptions in mind we can proceed with the discussion.

The incorporation of requirements into the network flow model will follow the approach used by other authors in formulating similar problems.[6,7] The idea is to use a valuing complex in the network which awards a profit for meeting each requirement. The maximization of this profit can be accomplished in the network flow model by letting profit be represented as a negative cost (i.e., $-C_{ij}$). It follows then that minimizing costs in the total network tends to maximize profit.

In the officer personnel planning problem we will first determine all possible billets to which an officer of a particular type could be assigned, then one node for each possible employment will be added to the network. Each such assignment would have a cost, C_{ij}, associated with it. This profit is defined to be the negative of the cost, C_{ij}, which would be assigned to the condition of not having the i^{th} type of officer available to fill the j^{th} type billet. For example, in terms of

Figure 2, if we were considering the billet which required an officer whose qualifications corresponded to the four year point of Figure 2, then minus C_{ij} would be minus [f(4) plus the maintenance cost associated with that type of officer]. We then capacitate these profit arcs by setting $L_{ij} = 0$ and M_{ij} equal to the requirement for the i^{th} type of officer in the j^{th} type billet. This device can be used to incorporate all the given requirements into the model.

4.3. Gains, Losses and Initial Inputs. Gains and initial inputs both represent flows from the source into the network. Initial inputs are used only to provide the model with the starting conditions (i.e., how many officers are there of each type at time zero). Each input is made by drawing an arc from the source to the proper node and setting the arc lower bound equal to the arc upper bound equal to the desired input. According to Gorham, all inputs can be made in this way.[6] Gains then represent flows into the system after time zero and are accommodated by drawing an arc from the source to the desired node. Losses represent flows out of the system and are incorporated by drawing an arc from the appropriate node to the sink.

In general, it is appropriate and convenient to assume that all gains and losses occur at the same instant for a given time period. Gains are primarily made at only a few points corresponding to recruiting inputs; of course, gains can also be made at the proper points to reflect reserve recall or other procurement policies. Gain arcs will have zero costs, zero lower bounds, and finite positive upper bounds as desired. Loss arcs are more troublesome because they can occur at many points in the model and they are usually voluntary. The latter fact can be simulated by making projections of losses based on past experience and setting fixed loss flows into the model. As in the case of gain arcs, we will have zero costs associated with loss arcs. The fact that losses can occur at many points is no problem if the career states are chosen in a manner which facilitates computation of a loss projection for that career state.

4.4. Network Synthesis. The actual network which would be developed to represent the officer personnel system would depend upon the definition and number of career states used, the way in which requirements and losses are given and the constraints, policies, and regulations to be considered. Actual network design would probably be unique for a given set of conditions so it is not possible to present a typical network, nor can the discussion be completely general and still be meaningful.

Before we proceed to illustrate how a network could be constructed, two important general conditions must be discussed. It will be recalled that we indicated in Figure 1 that we would be planning for a time horizon "T" that included more than one time period. Also recall from Figure 6 that more than one type of officer could be assigned to the same type of billet. These two facts are in apparent conflict with our method of including node capacity constraints because if we assign different types of officers to a particular type of billet in time period 1, then there is no easy way to separate the flow out of that node (which represented the type of billet) into the types of officers which were assigned. This means that we could not subsequently identify a particular arc with a particular type of officer and, therefore, could no longer control the number of officers of that type. We can avoid mixing the various ranks in our model by making parallel paths through the system; one path for each type of officer. By doing this, we can gather all of the arcs containing a particular type of officer at a constrained node once during each time period and thereby include our node capacity type constraints.

The solution proposed in the prior paragraph introduces the second condition alluded to earlier. Since, by hypothesis, we can, in general, assign more than one type of officer to a particular billet, we would expect to find nodes representing that billet in more than one of the parallel paths through the system. We would not want to award a profit for filling the same requirement once with one type of officer in one path

and then again for filling the same requirement with an officer of a different type in a different path. This means that we must a priori assign upper bounds to the profit arcs of each path such that the upper bounds sum to the given requirement with respect to a particular billet. This allocation of requirements should be done in a manner which will maximize planning effectiveness. This implies that we must in effect solve our network flow model in two stages. First, we are allocating our resources in an optimum manner under the assumption that such resources are available, and second, we must determine the flows that will provide those resources. The first stage is accomplished by solving the following linear program:

$$\text{Minimize} \quad \sum_{t=1}^{T} \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ijt} f_{ijt}$$

$$\text{Subject to} \quad f_{ijt} \geq 0 \qquad \text{all } i, j, t$$

$$\sum_{j=1}^{n} f_{ij1} \leq a_{i1} \qquad i = 1, 2, \ldots, m$$

$$\sum_{j=1}^{n} f_{ij2} \leq a_{i2} \qquad i = 1, 2, \ldots, m$$

$$\vdots$$

$$\sum_{j=1}^{n} f_{ijT} \leq a_{iT} \qquad i = 1, 2, \ldots, m$$

and

$$\sum_{i=1}^{m} f_{ij1} \leq r_{j1} \qquad j = 1, 2, \ldots, n$$

$$\sum_{i=1}^{m} f_{ij2} \leq r_{j2} \qquad j = 1, 2, \ldots, n$$

$$\vdots$$

$$\sum_{i=1}^{m} f_{ijT} \leq r_{jT} \qquad j = 1, 2, \ldots, n$$

Clearly this is equivalent to working the sequence of problems:

$$\text{Minimize} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ijt} f_{ijt} \qquad t = 1, 2, \ldots, T$$

$$\text{Subject to} \quad \sum_{j=1}^{n} f_{ijt} \le a_{it} \qquad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} f_{ijt} \le r_{jt} \qquad j = 1, 2, \ldots, n$$

$$\text{and} \quad f_{ijt} \ge 0 \qquad \text{for all } i, j, t$$

where a_{it} is the constraint on the total number of i^{th} type officers in year t; r_{jt} is the given requirement for the j^{th} type billet in the t^{th} year; C_{ijt} is the penalty cost plus maintenance cost associated with assigning the i^{th} type officer to the j^{th} billet in time period t; and f_{ijt} is the number of officers of the i^{th} career state assigned to the j^{th} billet during time period t. We have elected to use the latter linear programs in this thesis because each subprogram can be easily solved with the Out-of-Kilter algorithm.[12] The elements of the solution vectors, f_{ijt}, become the upper bounds, M_{ij}, for the profit arcs in the network flow model which represents the entire personnel system.

We can now illustrate the use of the foregoing concepts and give an intuitive indication of what a complete system model might look like. We start the problem with an on-board population which is first partitioned into career states of interest. Then the constraints must be identified and analyzed to determine the allowable and required movements (arcs) between nodes. Next the position of gain and loss arcs should be determined and these arcs added. Then identify the requirements, construct and add the valuing complex. Figure 7, below, illustrates what the path for <u>one type of officer during one time period</u> would look like:

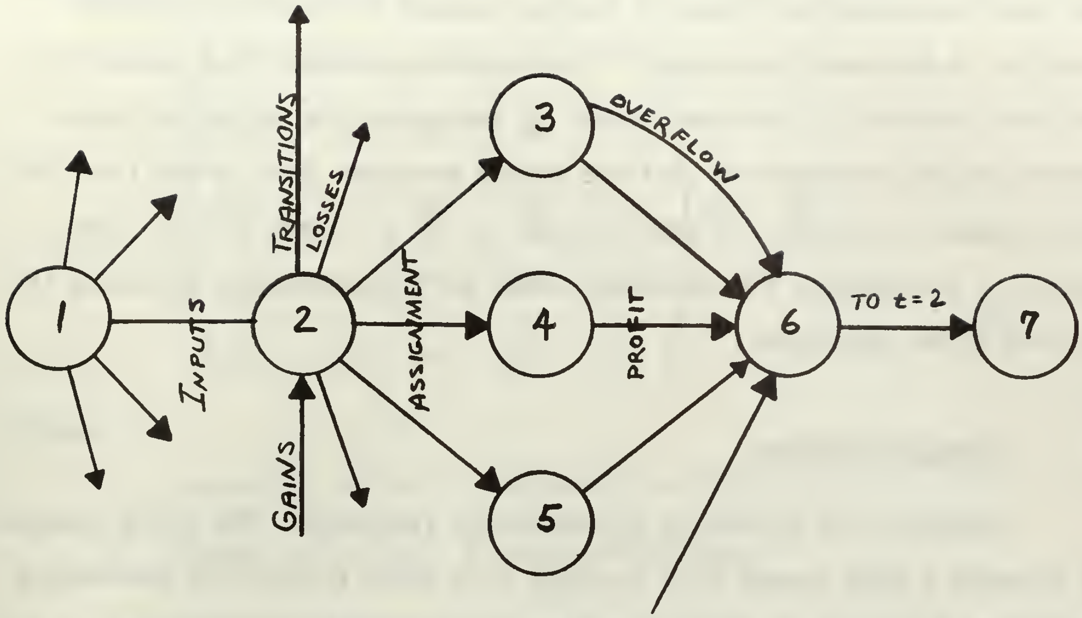


FIGURE 7

With reference to Figure 7, arcs originating at node 1 (the source) represent the system inputs at time zero. In particular, the flow over arc (1,2) is the starting number of officers of the particular type at which we are looking. All gains and losses during the period from $t = 0$ to $t = 1$ take place at node 2. Arcs (2,3), (2,4), and (2,5) are assignment arcs. These arcs have costs C_{23}, C_{24}, C_{25} assigned which are the costs associated with assigning an officer from node 2, to the billets represented by nodes 3, 4, and 5. The overflow arc from node 3 to node 6 is included to allow for the possibility of having more officers of this type than needed during this time period. A flow would occur in this arc only if the model required the extra officers of this type in a subsequent time period or if the node 2 gain and loss arcs had constraints which forced an overflow situation. Arcs (3,6), (4,6), and (5,6) are the valuing complex and have the negative cost figure assigned which represents the profit for meeting a requirement. These arcs have upper bounds which are calculated by solving the linear program discussed earlier in this section. Arc (6,7) is then the input arc for the next time period, and node 7 is the time period two analog of node 2.

As implied by Figure 7, the complete system could be graphed for one time period by drawing similar graphs for each type officer parallel to the one illustrated. The complete network flow model for the time horizon T will be formed by stringing the individual time-period graphs together so that the output from the first is the input for the second, etc., for all time periods t in T. After all L_{ij}, M_{ij}, and C_{ij} are added, the model is ready for final solution by using the Out-of-Kilter algorithm.

5. Example Problem.

Consider the following hypothetical situation: The Navy intends to develop a high speed ASW surface ship force which will commence personnel build-up at $t = 0$ and attain full operating strength at $t = 5$. We know how many officer personnel are required to man the force and are asked to plan for the orderly and economical build-up in personnel requirements of this force. The assumed requirements, inputs, and system parameters are as given in the following paragraph.

There are five types of billets to be filled; these are designated A, B, C, D, and E. We have five types of officers as follows:

Commander (CDR)

Lieutenant Commander (LCDR)

Lieutenant (LT)

Lieutenant Junior Grade (LTJG)

Ensign (ENS)

We start the problem with the following on-board vector:

$Y = (10, 10, 30, 40, 40)$ where

$y_1 = 10$ CDRs

$y_2 = 10$ LCDRs

$y_3 = 30$ LTs

$y_4 = 40$ LTJGs

$y_5 = 40$ ENSs

Our expected requirements matrix for the five time periods is as follows:

$$\begin{pmatrix} 10 & 15 & 20 & 30 & 30 \\ 10 & 15 & 20 & 30 & 30 \\ 30 & 45 & 60 & 90 & 90 \\ 40 & 60 & 80 & 120 & 120 \\ 40 & 60 & 80 & 120 & 120 \end{pmatrix} = (r_{jt})$$

where

r_{1t} number of billets A required at time t

r_{2t} number of billets B required at time t

r_{3t} number of billets C required at time t

r_{4t} number of billets D required at time t

r_{5t} number of billets E required at time t

We expect to lose a certain number of officers of each type during each time period t. These expected losses are as follows:

	Time Period 1	2	3	4	5
CDR	3	3	6	9	9
LCDR	1	1	2	3	3
LT	2	2	4	6	6
LTJG	20	30	40	60	60
ENS	6	9	12	18	18

In a problem of this sort we have to assume that constraints on officer type totals exist due to effective policy decisions outside the purview of the model. These are as follow:

	Time Period				
	1	2	3	4	5
CDR	9	14	20	33	33
LCDR	8	12	18	30	33
LT	33	50	66	99	99
LTJG	45	69	90	132	132
ENS	45	75	95	135	132

Certain personnel policies are in effect (or it is desired that they be in effect) which will have a bearing on the model solution; these are discussed in the following paragraphs.

The assignment policies for the entire planning horizon are:

CDRs	to billets A or B
LCDRs	to billets A, B, or C
LTs	to billets B, C, or D
LTJGs	to billets C, D, or E
ENSs	to billets D or E

This information permits derivation of the penalty cost plus maintenance cost matrix, C. The basis for cost information used and the complete derivation of the matrix C (C_{ij}) is contained in Appendix II. The matrix C is:

$$\begin{pmatrix} 17 & 108 & & & \\ 106 & 15 & 85 & & \\ & 84 & 14 & 45 & \\ & & 43 & 12 & 29 \\ & & & 27 & 10 \\ 246 & 153 & 82 & 49 & 30 \end{pmatrix}$$

(The elements C_{ij} are in thousands of dollars)

Inputs may be made to the ENS career states without penalty cost or constraint. Inputs may also be made to the other states at any time and in any amount, but because this action will have an adverse impact

on other naval programs and because considerable special training will be required to make up for the lack of proper experience, we are given the following penalty costs which apply to such direct inputs:

	Input Year 1	2	3	4	5
CDR	150	175	200	225	250
LCDR	95	105	120	135	165
LT	35	45	55	65	90
LTJG	15	20	25	30	35

(The penalty costs are in thousands of dollars)

Promotion policies are such that promotions are asumed to occur on demand subject to the following constraints:

	Time Period 1	2	3	4	5
LCDR to CDR	3	4	5	8	9
LT to LCDR	6	9	12	18	18
LTJG to LT	13	20	26	40	40
ENS to LTJG	26	40	52	80	80

The entries in the body of the table represent the maximum numbers which can be promoted due to minimum time in grade limitations, promotion rates, etc. This promotion scheme follows from the fact that in this example we are optimizing a program within a larger system. Promotions take place on a Navy-wide basis and might or might not fit the promotional requirements of this program. The given promotion constraints represent an estimate of the Navy-wide promotion rates, as applied to the numbers in each state. Obviously, if the model requires more promotions than we allow, provision must be made for other inputs to make up for the deficiency; this has been done in the example problem.

On the other hand, if the model requirements were exceeded by the promotion flows prevailing, then we would have to provide a means of transferring this excess out of the model. This possibility has been ignored in the example problem.

With the foregoing information, we are ready to formulate the network under the general assumptions given in Chapters 3 and 4. First consider the CDR path for one time period; the graph is

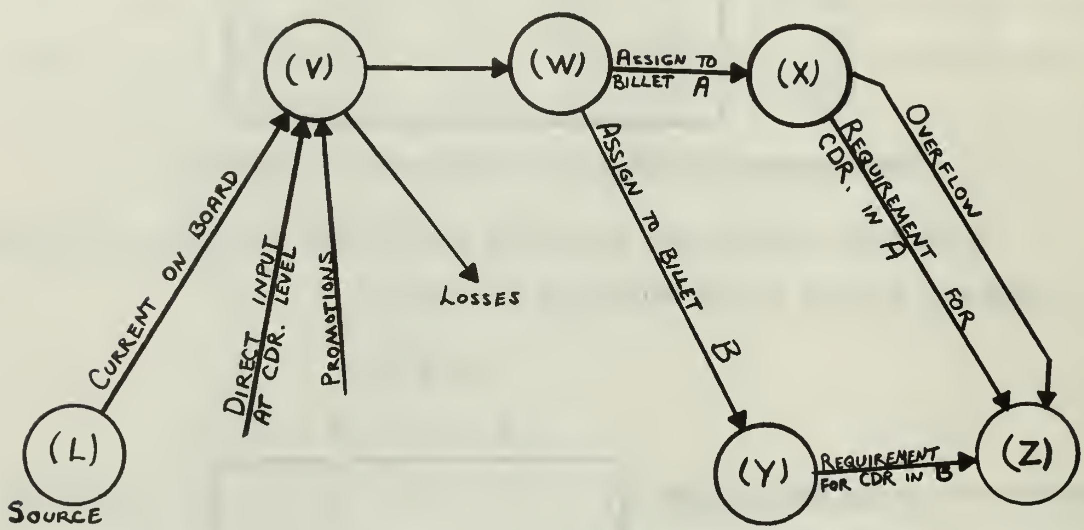
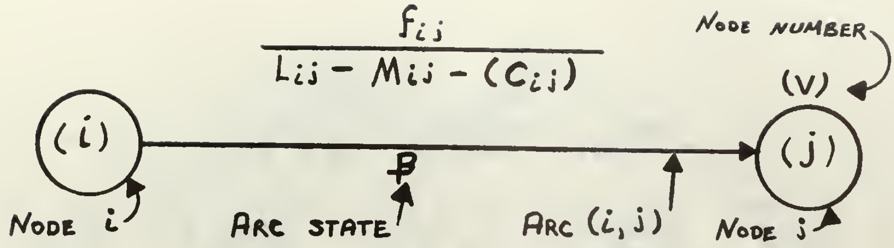


FIGURE 8

Before we can draw the network we must calculate the optimum upper bounds for the profit arcs of the assignment phases of the problem (i.e., arcs (x,z) and (y,z) in Figure 8, above). This will require the solution of five network flow problems. The five all have the same graph of arcs and nodes; only the numbers change from time period to time period. The network flow problem for time period one is shown below (Figure 9) along with the solution flows and node numbers (v). The notation employed in Figure 9 is as follows:



It can also be shown that the solutions to the other four problems are as given in the following tableau. (Note: the entries in the body of the tableau become the upper bounds on the appropriate arcs of the complete network.)

	Time Period 1	2	3	4	5
CDR to A	9	14	20	30	30
CDR to B	0	0	0	0	0
LCDR to A	1	1	0	0	0
LCDR to B	7	11	18	30	30
LCDR to C	0	0	0	0	0
LT to B	3	4	2	0	0
LT to C	30	45	60	90	90
LT to D	0	0	0	0	0
LTJG to C	0	0	0	0	0
LTJG to D	40	60	80	120	120
LTJG to E	0	0	0	0	0
ENS to D	0	0	0	0	0
ENS to E	40	60	80	120	120

We can now construct the complete network by repeating Figure 8 for the four successive time periods; then repeating the process for each of the other career states; then paralleling the five career state graphs, and finally adding the appropriate interconnecting arcs. Figure 10 shows how the network would look with all arcs and nodes combined; the relevant numbers have been omitted for clarity but are tabulated in Appendix III.

TIME PERIOD t = 1

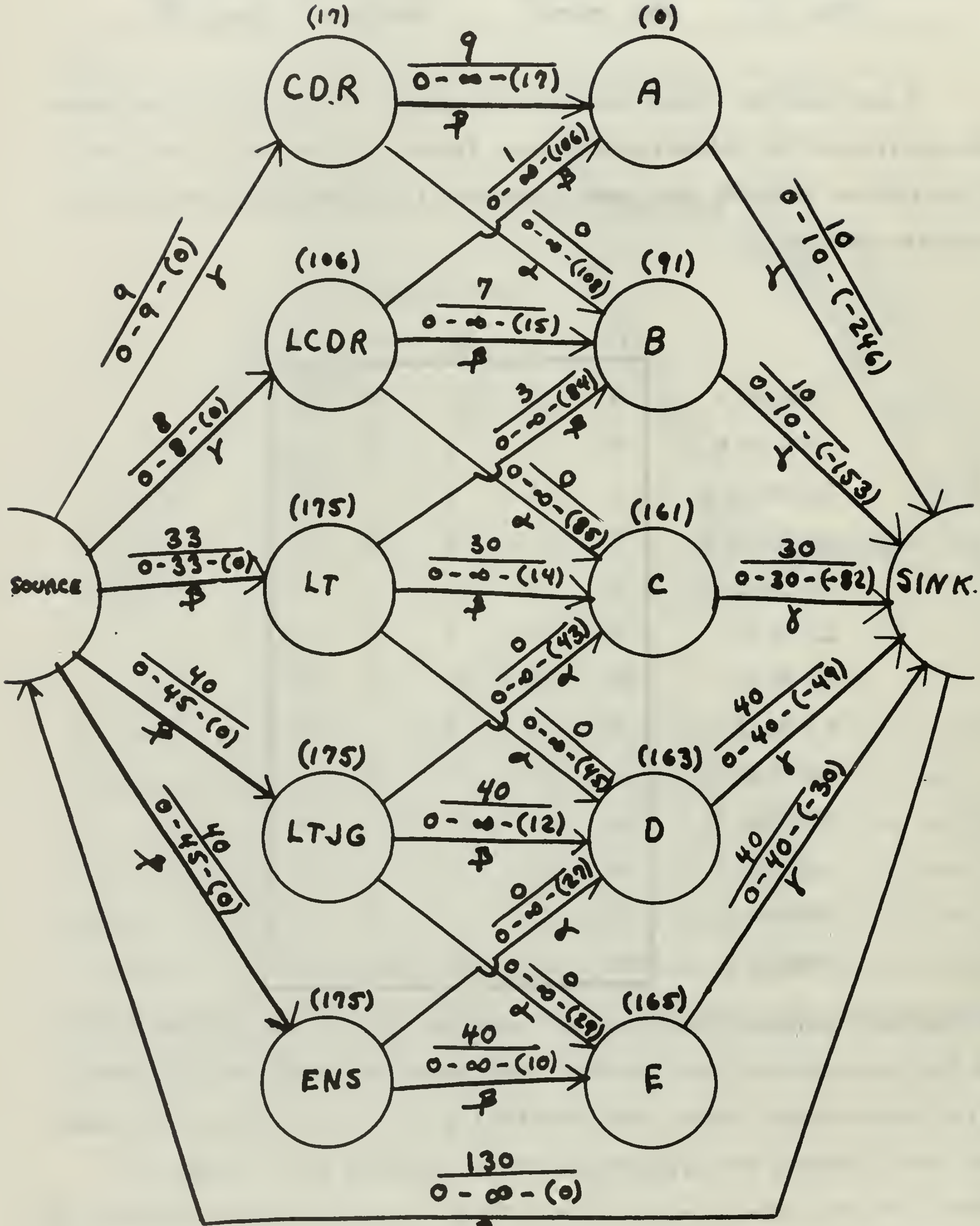


FIGURE 9

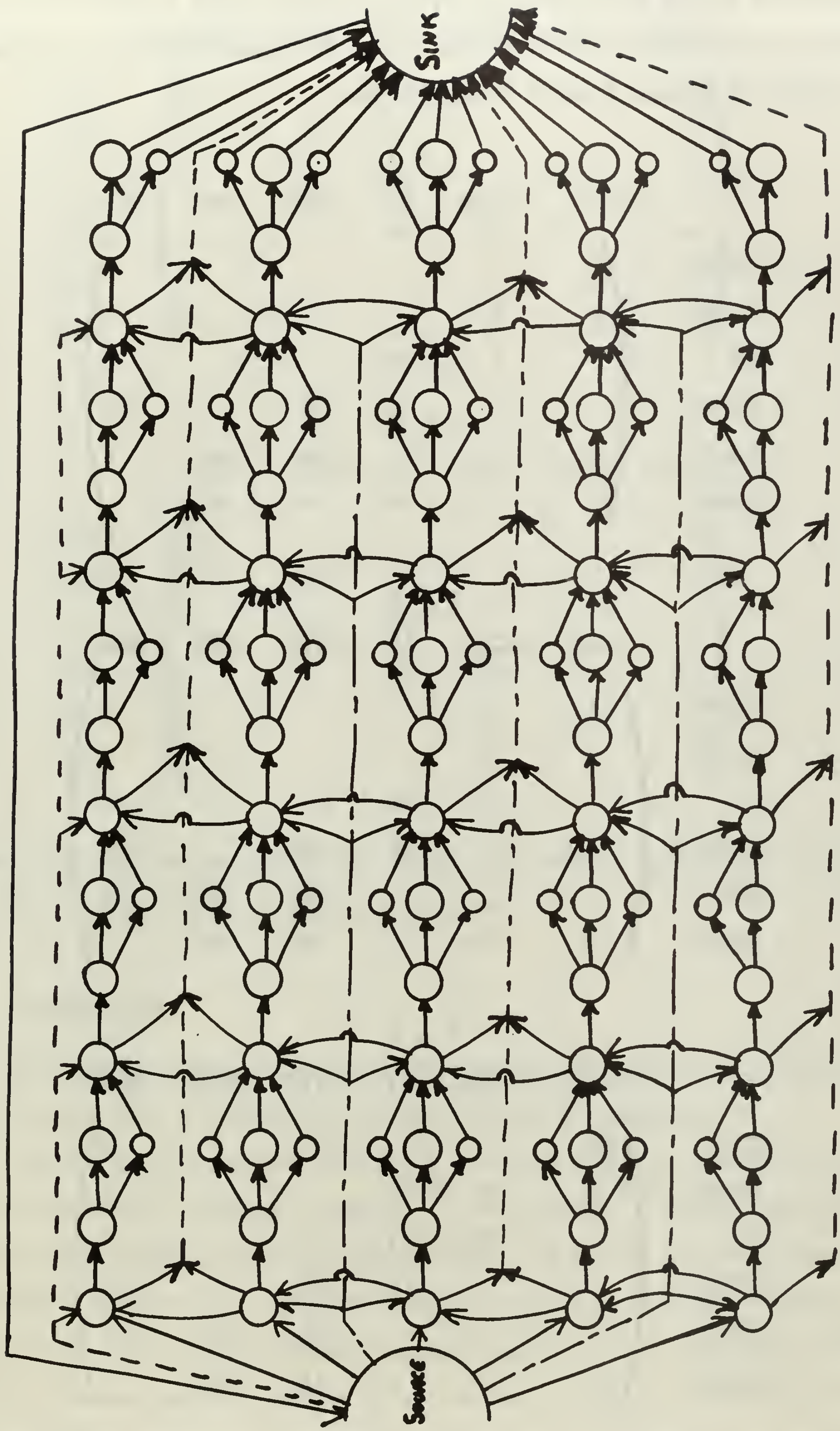


FIGURE 10

The network represented by Figure 10 can be solved to give the following set of flow matrices where the zero state is taken to be either the source or sink as appropriate. The complete solution to the problem is given in tabular form in Appendix III.

$$t = 1$$

$$\begin{array}{cc} & \begin{array}{cccccc} \text{CDR} & \text{LCDR} & \text{LT} & \text{LTJG} & \text{ENS} & \text{ZERO} \end{array} \\ \begin{array}{l} \text{CDR} \\ \text{LCDR} \\ \text{LT} \\ \text{LTJG} \\ \text{ENS} \\ \text{ZERO} \end{array} & \begin{pmatrix} 9 & & & & & 3 \\ 2 & 8 & & & & 1 \\ & 1 & 33 & & & 2 \\ & & 6 & 40 & & 20 \\ & & & 26 & 40 & 6 \\ 0 & 0 & 0 & 0 & 32 & \end{pmatrix} \end{array}$$

$$t = 2$$

$$\begin{array}{cc} & \begin{array}{cccccc} \text{CDR} & \text{LCDR} & \text{LT} & \text{LTJG} & \text{ENS} & \text{ZERO} \end{array} \\ \begin{array}{l} \text{CDR} \\ \text{LCDR} \\ \text{LT} \\ \text{LTJG} \\ \text{ENS} \\ \text{ZERO} \end{array} & \begin{pmatrix} 14 & & & & & 3 \\ 4 & 12 & & & & 1 \\ & 9 & 49 & & & 2 \\ & & 20 & 60 & & 30 \\ & & & 40 & 60 & 9 \\ 4 & 0 & 7 & 30 & 69 & \end{pmatrix} \end{array}$$

$$t = 3$$

$$\begin{array}{cc} & \begin{array}{cccccc} \text{CDR} & \text{LCDR} & \text{LT} & \text{LTJG} & \text{ENS} & \text{ZERO} \end{array} \\ \begin{array}{l} \text{CDR} \\ \text{LCDR} \\ \text{LT} \\ \text{LTJG} \\ \text{ENS} \\ \text{ZERO} \end{array} & \begin{pmatrix} 20 & & & & & 6 \\ 5 & 18 & & & & 2 \\ & 12 & 62 & & & 4 \\ & & 26 & 80 & & 40 \\ & & & 52 & 80 & 12 \\ 7 & 1 & 3 & 34 & 84 & \end{pmatrix} \end{array}$$

$$t = 4$$

$$\begin{array}{c|cccccc} & \text{CDR} & \text{LCDR} & \text{LT} & \text{LTJG} & \text{ENS} & \text{ZERO} \\ \hline \text{CDR} & 30 & & & & & 9 \\ \text{LCDR} & 8 & 30 & & & & 3 \\ \text{LT} & & 18 & 90 & & & 6 \\ \text{LTJG} & & & 40 & 120 & & 60 \\ \text{ENS} & & & & 80 & 120 & 18 \\ \text{ZERO} & 11 & 5 & 12 & 60 & 138 & \end{array}$$

$$t = 5$$

$$\begin{array}{c|cccccc} & \text{CDR} & \text{LCDR} & \text{LT} & \text{LTJG} & \text{ENS} & \text{ZERO} \\ \hline \text{CDR} & 30 & & & & & 9 \\ \text{LCDR} & 9 & 30 & & & & 3 \\ \text{LT} & & 12 & 90 & & & 6 \\ \text{LTJG} & & & 18 & 120 & & 60 \\ \text{ENS} & & & & 78 & 120 & 18 \\ \text{ZERO} & 0 & 0 & 0 & 0 & 96 & \end{array}$$

These five matrices represent the flows that will be required to meet the given requirements in optimum fashion.

6. Discussion.

Certain features of the model and its formulation require amplification; these will be discussed in the following paragraphs.

Career states may be defined in any way that the personnel manager finds meaningful; however, Merick[13] and Gorham[6] have noted two points which should be considered. First, the career states defined should be homogeneous. If distinctions must be considered within a career state, then the state must be fragmented. It is, however, desirable to keep the basic state as inclusive as possible because the time and effort required to compute a solution is related to the number

of states considered. Second, the partition of the naval officer population should not be too fine because the number of officers in a given state would be too small to have any statistical significance. This becomes important when we try to determine loss rates to use in the model. Recalling that loss flows are to be set into the model, we can see that if the partition were such that some career states contained only 2 or 3 officers, it would be very difficult to accurately estimate what loss flows to use.

In the derivation of a measure of effectiveness we were careful to avoid the inclusion of qualitative measures in the discussion. This is because the problem we seek to solve is a planning problem and not an assignment problem. It would be impossible to assume or infer a priori that officers of one career state would be better performers or more highly motivated than those of some other career state. The many qualitative factors that so influence the assignment problem are always obtained from the individual's record rather than from his career state and, hence, are not appropriate in the planning context.

The same type of argument can be applied with regard to the handling of over-qualified officers in this problem. If we were talking about distribution, we would look at over-qualification as a sunk cost and assign as though the over-qualification did not exist; however, within the planning context, over-qualification is as much a failure as under-qualification since either situation may be rectified by the appropriate planning changes. So long as our resource constraints are binding, it follows that surplus in one area implies shortage in another; hence, we could not in general attach different measures to the conditions of over-qualification or under-qualification.

Five significant assumptions have been used in the thesis. These are summarized as follows:

(1) requirements are given to the personnel planner;

(2) the amount of carry-down is specified; that is, the numerical constraints on types of officers are given;

(3) it will not always be possible to program a correctly qualified individual for each requirement;

(4) the limitations given in assumption 2 are the binding constraints in the model; and

(5) changes in constraints, costs, and/or requirements occur simultaneously once each time period.

The first three assumptions were made to define the problem. They are considered to be realistic in that either they reflect existing conditions or such conditions could be expected to exist if such a model were actually constructed. The fourth assumption followed directly from current distribution policies which were discussed in Section 4.2. This is the key assumption which allows this formulation to cover multiple time periods in one model. If this assumption were not true, then it would not be possible to logically allocate the requirements among the various paths of the network. This would in turn force the network into a single time period formulation. Of course the model could be solved recursively, one time period at a time, but it is not clear that this procedure would yield an optimum solution to the planning problem. The fifth assumption was made primarily for convenience in network construction; it does not detract from the model's generality, but does simplify the network.

The Out-of-Kilter algorithm was selected for use in the model for several reasons.[6,12] First, the algorithm is readily adaptable to computer solution and can therefore handle a relatively large number of nodes and arcs. Second, the algorithm is efficient in the sense that the solution is never worsened at any step, a fact that complements computer computational methods. Third, it is well suited to personnel problems in that if the upper and lower bounds on all arcs are integer, then the solution flows will be integer. Fourth, the solution procedure may be started with any circulation where a circulation is defined to be a set of network flows such that $\sum_j (f_{ij} - f_{ji}) = 0$ for all nodes in the network. It is therefore easy to obtain a starting

circulation for a computer program; i.e., all $f_{ij} = 0$. Additionally, it should be obvious that once a solution has been obtained, we can alter L_{ij}, M_{ij}, or C_{ij} without disturbing the solution circulation and thereby still maintain a starting flow to solve the modified network with a minimum amount of computational effort. This facilitates parametric study of constraints and costs in the model.

The implementation of a network flow planning model, such as has been described in this thesis, can be expected to yield several advantages. First, it is possible to simultaneously consider the total personnel system for a reasonable number of time periods, thereby allowing the many requirements, constraints, and policies to properly interact. Second, the model will pinpoint problem areas and assist in locating the planning correction needed. Third, the model can be used for parametric analysis of personnel policies used in the model. For example, in the sample problem of Section 5, after we achieved the first solution, we could have asked and answered such questions as: What will happen if we do not allow direct input at the CDR level in year 5? Or, what will be the effect of increasing promotion rates from LT to LCDR after year 2? Obviously we could compile a huge list of such questions. We can answer such questions by making the desired changes in L_{ij}, C_{ij}, and/or M_{ij}; then apply the Out-of-Kilter algorithm to bring the affected arcs back into kilter. Fourth, the model can be rerun frequently with minor network modifications as changes in the system occur.

By this time the limitations or drawbacks of the proposed model are probably also apparent. The first is that an enormous amount of effort would be required to develop and program the first model. The effort involved in estimating costs and computing the C_{ij} alone would be substantial.

The second possible drawback concerns the question of computational feasibility. It should be fairly obvious that a network flow model

of the entire naval officer personnel system could assume gigantic proportions; it could also be quite small. Models that are too large make solution so difficult that the appeal of the approach is lost, while on the other hand a model consisting of only two career states would hardly provide any meaningful information. Obviously, the goal is to find that in-between size that provides useful assistance to the personnel planner yet is not so big that computer facilities are strained or exceeded. At the present time, a ready-made program utilizing the Out-of-Kilter algorithm is available from the SHARE distribution agency for the IBM 7090 computer. This particular program (RSOKF1) will accommodate 1500 nodes and 4500 arcs. The network flow model which has been proposed herein is estimated to require approximately five nodes per career state per time period. If five time-period studies are desired, then a network flow model for sixty states should result in about 1500 nodes and 4000 arcs. Such a model could be handled by the existing program. It is not possible to predict run time as a direct function of the number of nodes and arcs since the number of computations will vary substantially from problem to problem; however, as an example, a problem consisting of 777 nodes and 2899 arcs required 1139 breakthroughs and 411 non-breakthroughs to achieve solution on an IBM 7090. That problem required 5 minutes of computer time exclusive of an input and output time of about 3 minutes.[12] It seems likely that the model described in this paper with its low ratio of arcs to nodes (less than 3 to 1) would not take any more time, for an equivalent sized model, than the example cited. In fact, experience with the hand computations involved in the example of the last section would indicate that this model is a fairly simple type to solve and would therefore require even less computer time than implied above. However, it is not known whether or not a 60 career state network would be of sufficient usefulness to justify its construction. If a larger network were desired, then a larger program and computer would

be required to solve the problem. This fact could limit the application of this particular network formulation.

7. Summary, Conclusions, and Recommendations.

The problem we started out to solve was given as: How can Navy personnel planners best meet expected future requirements for officer personnel with available resources and within existing legal and administrative constraints? We first hypothesized that the naval officer personnel system could be represented by a model consisting of nodes and arcs with flows moving between nodes of the system. Under this hypothesis, we reduced the planning problem to one of finding the set of flows that would move through our network when all requirements and constraints were satisfied.

At this point we assumed that the personnel planner might not always have sufficient numbers of fully qualified officers available to meet requirements. That assumption pointed up the need for a measure of planning effectiveness. This measure was then developed through appeal to the principle of revealed preference and took the form of penalty costs which were applied to those arcs of the model that assigned officers to billets. The minimum penalty costs were found to occur when an officer was assigned to the billet for which he was most qualified. Minimizing total penalty costs was shown to be the same as maximizing planning effectiveness.

We then proceeded to show that we could state the problem of finding a feasible flow through our network at minimum cost as a linear program for which a convenient solution algorithm was available. Section 4 was devoted to a discussion of how a network should be constructed if it were to be a valid representation of the officer personnel system through successive time periods. It was necessary to make certain additional assumptions during this development. In essence, these were:

(1) Gains and losses to the system simultaneously take place only once during each time period.

(2) The binding constraints on the total number of officers in each state can be identified.

(3) Requirements and constraints are given inputs to the planning problem.

Section 5 was devoted to demonstrating by example how the arguments of the preceeding sections could be applied to the task of solving a hypothetical personnel planning problem.

This thesis has shown that it is theoretically possible to represent a personnel system by a network flow model and that it is possible to solve such models for the minimum cost feasible flows. It has been further demonstrated that some naval officer personnel planning problems can be simulated by such a network. The most significant contribution of this thesis is considered to be the development of a measure of planning effectiveness which is closely related to the total utility of the personnel plan to the Navy. The second significant contribution is the fact that the model can simultaneously consider several time periods.

Since the model solution technique lends itself very admirably to parametric analysis of the many variables which influence the planning problem, we conclude that it should be used primarily to analyze the planning problem. A knowledgeable planning staff could, by studying the model behavior as a function of the inputs, determine the ranges of values over which changes in inputs were significant. With this information it should be possible to make valid predictions about the results to be expected from proposed changes in policy, inputs, attrition, etc. It should even be possible to make recommendations concerning needs for new policies or legislation when required.

The second conclusion is that the model will be most effectively used by persons who are knowledgeable about the personnel system.

It is not possible to design an all-purpose network; instead, the development of a network will be a unique evolutionary process. This process can best be accomplished by persons who know what the system really looks like. The guidelines in this thesis should be sufficient to draw a proper network which has a solution, but the actual fit of such a model to the true situation is a problem that can only be solved by the person constructing the network.

One major question remains unanswered at this time: that is, will the information provided by this technique be worth the time and effort which will be required to generate that information? It appears that this question can only be answered by making a test evaluation. To this end, it is recommended that further research be conducted in the form of an operational pilot model on some well defined segment of the naval officer population. As a suggestion, it would seem that the Supply Corps would be an appropriately sized group to work with. If such a pilot model were constructed and run, it should then be possible to make a firm recommendation regarding whether or not the method should be applied to the larger naval officer personnel planning problem.

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APPENDIX A

COMPUTATIONAL PROCEDURE[12]

The method presented here for computing optimal network flows has the following properties:

(a) Lower bounds as well as upper bounds are assumed for each arc flow; if not explicitly stated, we will assume a lower bound of zero and an upper bound of infinity.

(b) The cost coefficient for an arc is arbitrary in sign.

(c) The method can be initiated with any circulation, feasible or not, and any set of node numbers $\{v\}$.

The freedom to begin with any circulation and node numbers, instead of starting with particular ones which satisfy certain optimality properties, is perhaps the most important practical feature of the method. For example, in actual applications one is often interested in seeing what changes will occur in an optimal solution when some of the given data are altered. This method is tailored for such an examination, since the old optimal solution can be used to start the new problem, thereby greatly decreasing computation time.

Let v be any set of node numbers assigned to the network of interest (an easy starting point for computer solution is to take all node numbers equal to zero). Then we define:

$$\bar{a}(i,j) = a(i,j) + v(i) - v(j)$$

Then, for given v and circulation f, an arc (i,j) is in just one of the following states:

(α) $\bar{a}(i,j) > 0$, $f(i,j) = L(i,j)$

(β) $\bar{a}(i,j) = 0$, $L(i,j) \le f(i,j) \le C(i,j)$

(γ) $\bar{a}(i,j) < 0$, $f(i,j) = c(i,j)$

$(\alpha 1)$ $\bar{a}(i,j) > 0$, $f(i,j) < L(i,j)$

$(\beta 1)$ $\bar{a}(i,j) = 0$, $f(i,j) < L(i,j)$

$(\gamma 1)$ $\bar{a}(i,j) < 0$, $f(i,j) < c(i,j)$

$(\alpha 2)\ \bar{a}(i,j) > 0\ ,\quad f(i,j) > L(i,j)$

$(\beta 2)\ \bar{a}(i,j) = 0\ ,\quad f(i,j) > c(i,j)$

$(\gamma 2)\ \bar{a}(i,j) < 0\ ,\quad f(i,j) > c(i,j)$

We say that an arc is in kilter if it is in one of the states α, β, γ; otherwise the arc is out of kilter. Thus, to solve the problem it suffices to get all arcs in kilter. With each state that an arc (i,j) can be in, we associate a non-negative number, called the kilter number of the arc in the given state. An in-kilter arc has kilter number 0; the arc kilter numbers corresponding to each out-of-kilter state are listed below:

$(\alpha 1)$ or $(\beta 1)$: $L(i,j) - f(i,j)$

$(\gamma 1)$: $\bar{a}(i,j)[f(i,j) - c(i,j)]$

$(\alpha 2)$: $\bar{a}(i,j)[f(i,j) - L(i,j)]$

$(\beta 2)$ or $(\gamma 2)$: $f(i,j) - c(i,j)$

Thus out-of-kilter arcs have positive kilter numbers. The kilter numbers for states $\alpha 1$, $\beta 1$, $\beta 2$, $\gamma 2$ measure infeasibility for the arc flow f(i,j), while the kilter numbers for states $\gamma 1$, $\alpha 2$ are, in a sense, a measure of the degree to which the optimality properties fail to be satisfied.

The algorithm concentrates on a particular out-of-kilter arc and attempts to put it in kilter. It does this in such a way that all in-kilter arcs stay in kilter, whereas the kilter number for any out-of-kilter arc either decreases or stays the same. Thus, all arc kilter numbers are monotone non-increasing throughout the computation.

A basic notion underlying the method is to utilize the labeling process for increasing or decreasing a particular arc flow in a circulation.

<u>The out-of-kilter algorithm</u>. Enter with any integral circulation f and any set of node integers $\{v\}$. Next locate an out-of-kilter arc (s,t) and go on to the appropriate case below.

(s,t) is in state $(\alpha 1)$

Start a labeling process at t, trying to reach s, first assigning t the label $[s+, q(t) = L(s,t) - f(s,t)]$. The labeling rules are:

If node x is labeled $[z\pm, q(x)]$, node y is unlabeled, and if (x,y) is an arc in one of the states α, γ or $\gamma 1$, then node y receives the lable $[x+, q(y)]$, where:

$q(y) = \min\,[q(x), L(x,y) - f(x,y)]$ if (x,y) was in state $\alpha 1$;

$q(y) = \min\,[q(x), c(x,y) - f(x,y)]$ if (x,y) was in state γ or $\gamma 1$.

If node x is labeled $[z\pm, q(x)]$, node y is unlabeled, and if (y,x) is an arc in one of the states $\alpha 2$, β or $\gamma 2$, then y receives the label $[x-, q(y)]$, where:

$q(y) = \min\,[q(x), f(y,x) - L(y,x)]$ if (y,x) was in state $\alpha 2$ or β;

$q(y) = \min\,[q(x), f(y,x) - c(y,x)]$ if (y,x) was in state $\gamma 2$.

If breakthrough occurs (that is, s receives a label), so that a path from t to s has been found, change the circulation f by adding $q(s)$ to the flow in forward arcs of this path, subtracting $q(s)$ from the flow in reverse arcs, and finally adding $q(s)$ to $f(s,t)$. If non-breakthrough, let X and $\bar{X}$ denote labeled and unlabeled sets of nodes, and define two subsets of arcs:

$$A1 = \{(x,y) \mid x \text{ in } X,\ y \text{ in } \bar{X},\ \bar{a}(x,y) > 0,\ f(x,y) \le c(x,y)\}$$

$$A2 = \{(y,x) \mid x \text{ in } X,\ y \text{ in } \bar{X},\ \bar{a}(y,x) < 0,\ f(y,x) \ge L(y,x)\}$$

Then let

$$\delta 1 = \min_{A1} a(x,y)$$

$$\delta 2 = \min_{A2} -a(y,x)$$

$$\delta = \min(\delta 1, \delta 2)$$

(Here δi is a positive integer or ∞ according as Ai is non-empty or empty.) Change the node integers by adding δ to all $v(x)$ for x in $\bar{X}$.

(s,t) is in state $(\beta 1)$ or $(\gamma 1)$

Proceed the same as for arcs in state $(\alpha 1)$ except $q(t) = c(s,t) - f(s,t)$.

(s,t) is in state $(\alpha 2)$ or $(\beta 2)$

Here the labeling process starts at s, in an attempt to reach t. Node s is assigned the label $[t-, q(s) = f(s,t) - L(s,t)]$. The labeling rules are the same as were used for arcs in state $\alpha 1$. If breakthrough occurs, change the circulation by adding and subtracting $q(t)$ from the arc flows along the path from s to t; then subtract $q(t)$ from $f(s,t)$. If breakthrough does not occur, then change the node numbers as before.

(s,t) is in state $(\gamma 2)$

Proceed the same as for arcs in state $\alpha 2$ or $\beta 2$, except now $q(s) = f(s,t) - c(s,t)$.

The labeling process is repeated for the arc (s,t) until either (s,t) is in kilter or until a non-breakthrough occurs for which $\delta = \infty$. In the latter case, stop. (There is no feasible circulation.) In the former case, locate another out-of-kilter arc and continue.

It can be shown that the out-of-kilter algorithm terminates, and that all arc kilter numbers are monotone non-increasing throughout the computation.

APPENDIX B

DERIVATION OF THE PENALTY COST PLUS MAINTENANCE COST MATRIX

The cost figures used in this appendix have been extracted from a study by Robert J. Jackson, Lawrence O. Mann, Jr., and Walter H. Primes, Jr.[11] The report should be consulted for the various assumptions and estimating techniques used to derive these figures. They do not follow the exact procedure advocated in this thesis, but are considered to be satisfactory for example purposes. Costs in the approximation column are rounded off to the nearest thousand dollars to facilitate hand computational procedures.

Annual Maintenance Cost:

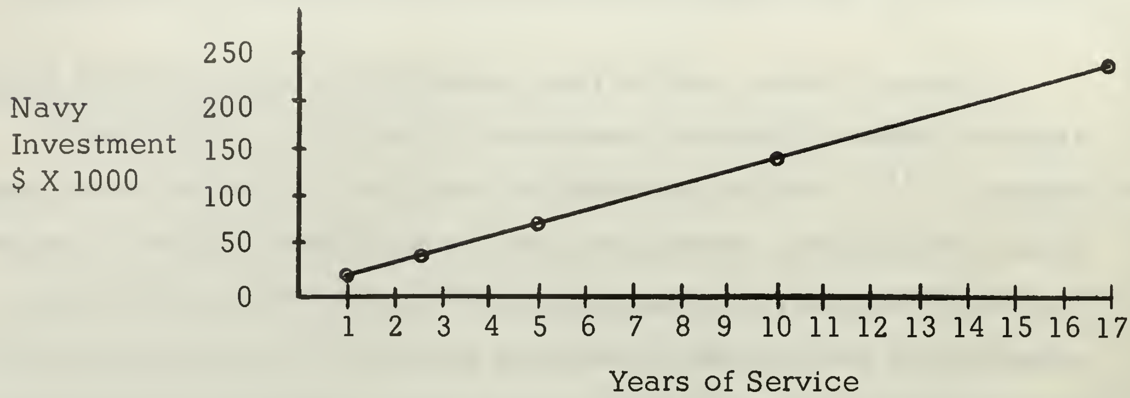
Type Officer	Exact Cost	Approximation
Ensign	9790.00	10 (x 1,000)
Lieutenant Junior Grade	11962.00	12
Lieutenant	13723.00	14
Lieutenant Commander	15058.00	15
Commander	16798.00	17

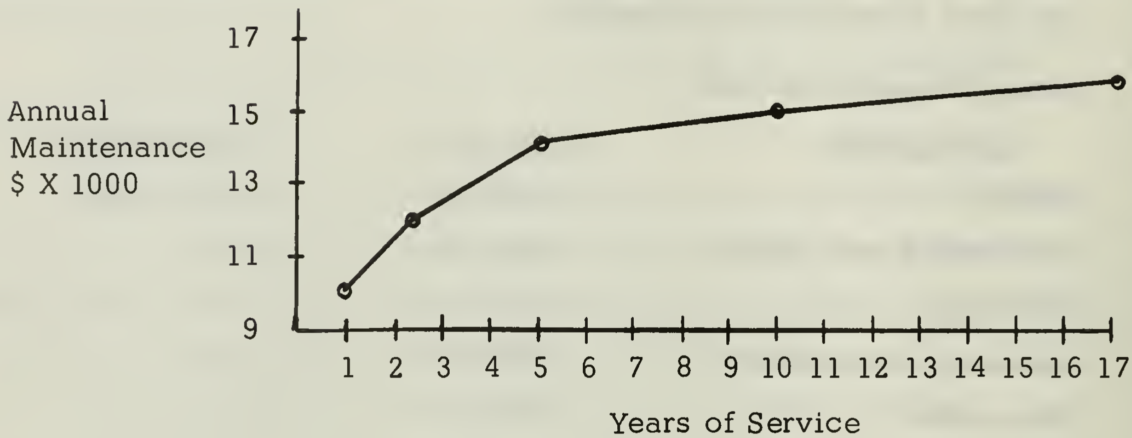
Investment Costs:

Type Officer	Exact Cost	Approximation
Ensign	19564.00	20 (x 1,000)
Lieutenant Junior Grade	37173.00	37
Lieutenant	67568.00	68
Lieutenant Commander	138263.00	138
Commander	229158.00	229

The matrix of penalty costs plus maintenance costs is derived as follows:

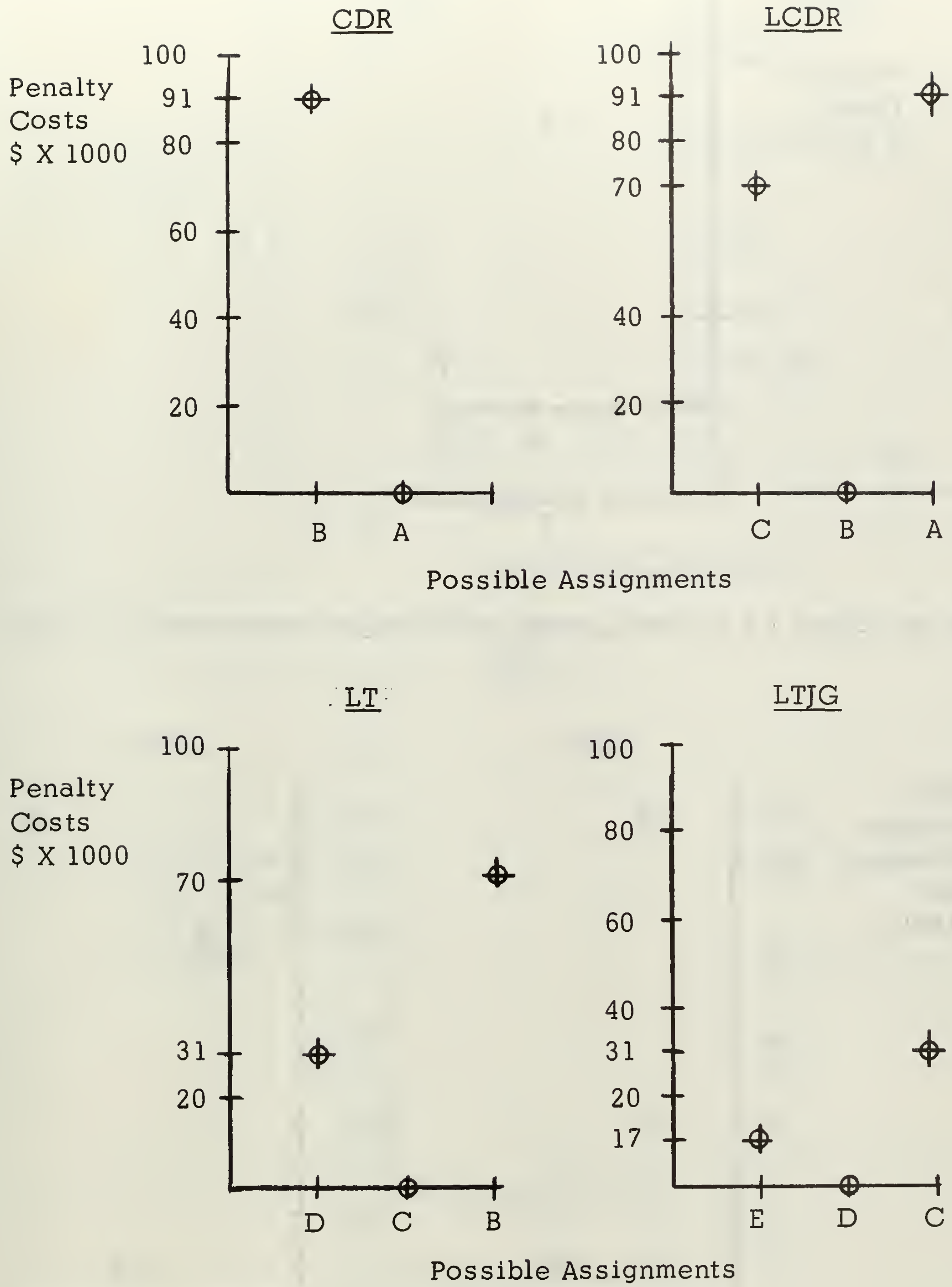
(1) First plot the given data.

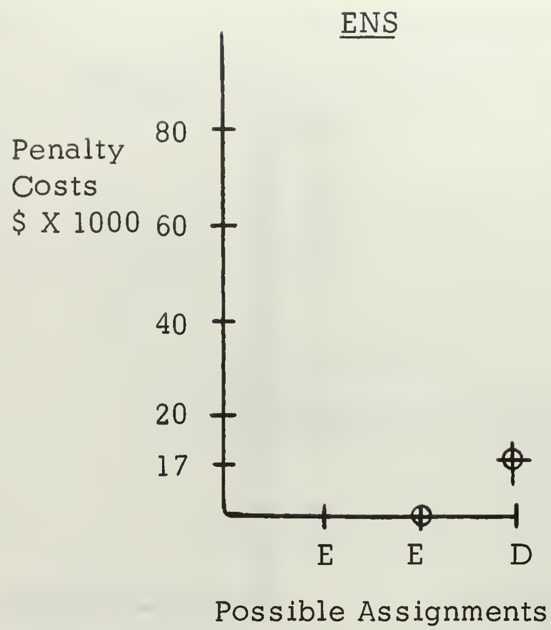




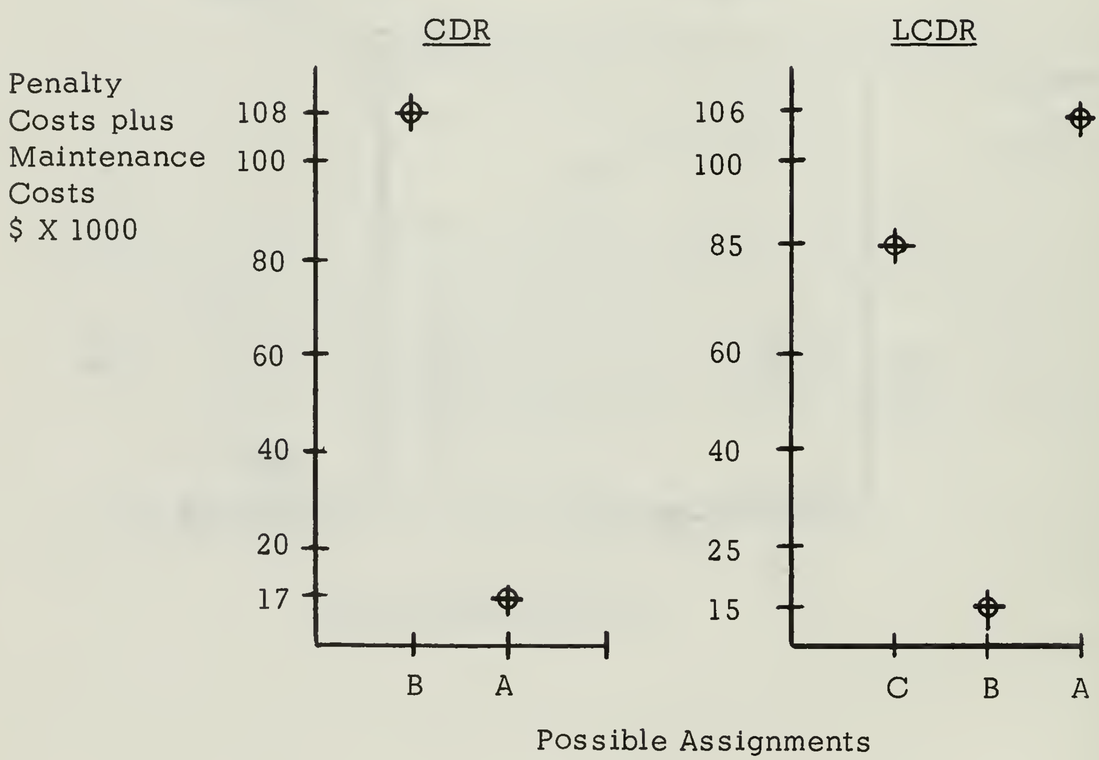
(2) For each officer we construct a Figure 4 and 5. The assignment policies are such that any officer may be assigned to a billet which requires an officer one rank senior to him and also to billets which require officers one rank junior to him.

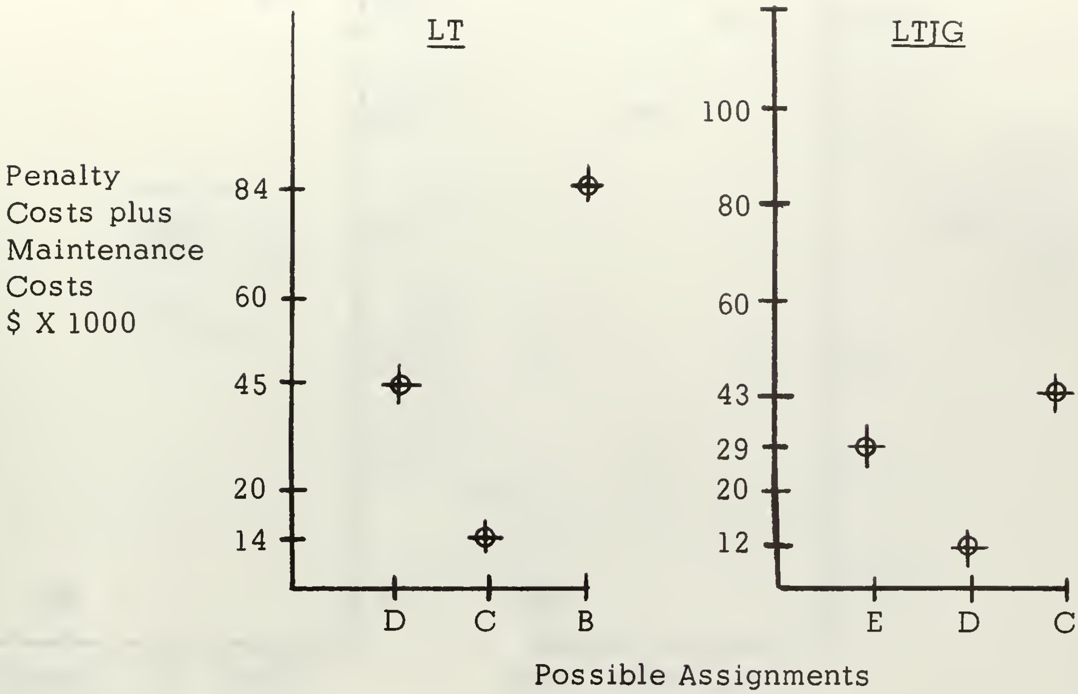
The applicable Figure 4's are as follows:

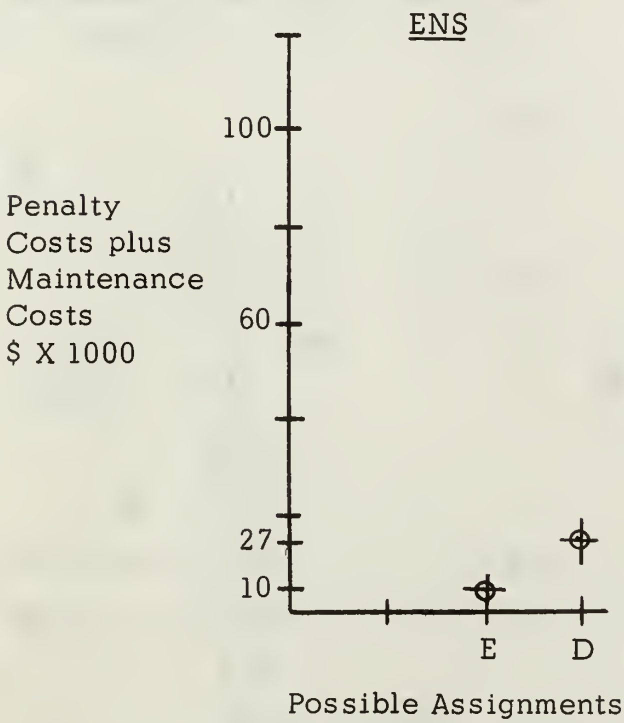

CDR
LCDR
Penalty Costs $ X 1000
100
91
80
70
60
40
20
B
A
C
Possible Assignments
LT
LTJG
Penalty Costs $ X 1000
100
80
70
60
40
31
20
17
D
C
B
E
Possible Assignments



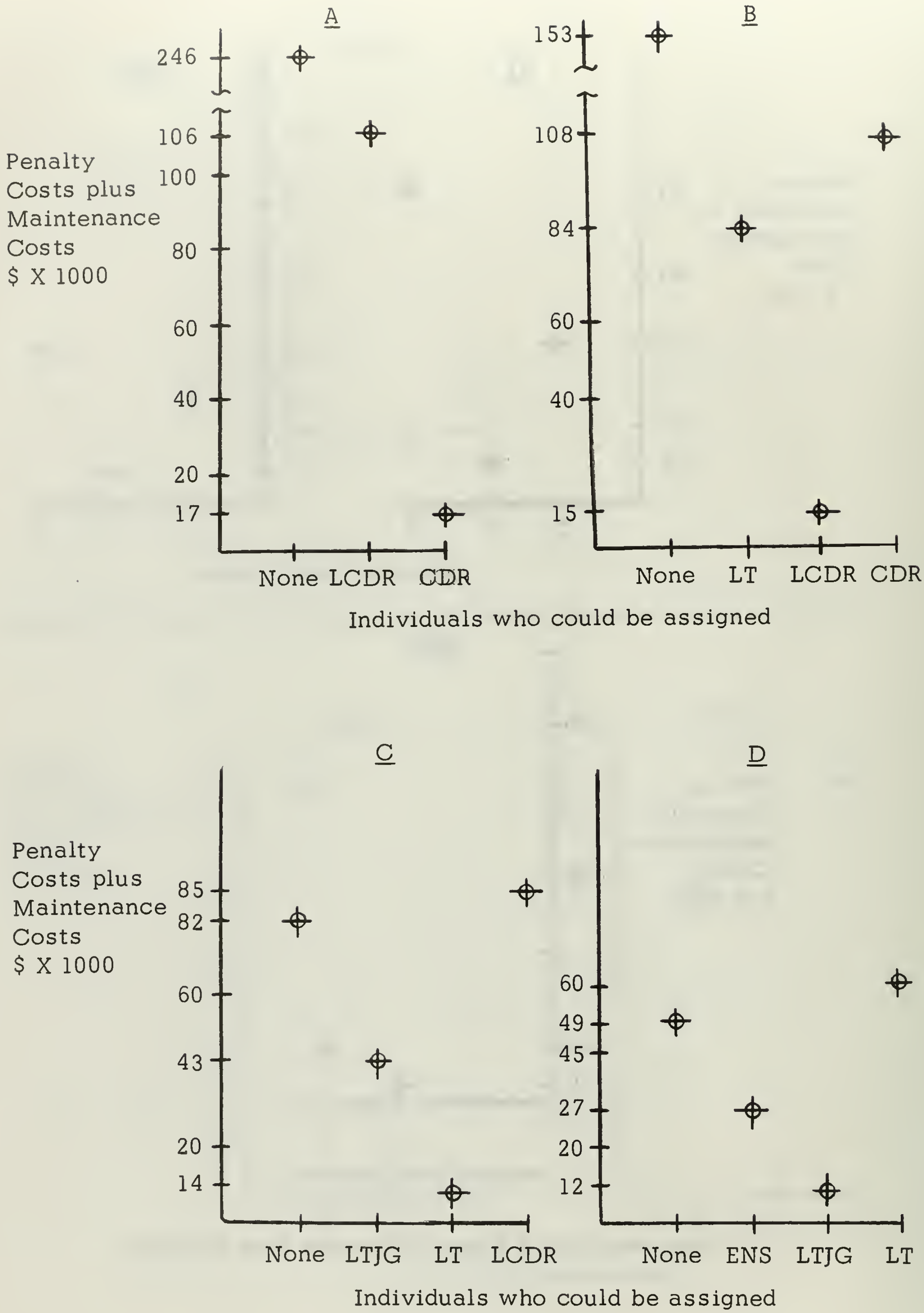
Then the Figure 5's for each career state can be constructed as follows:

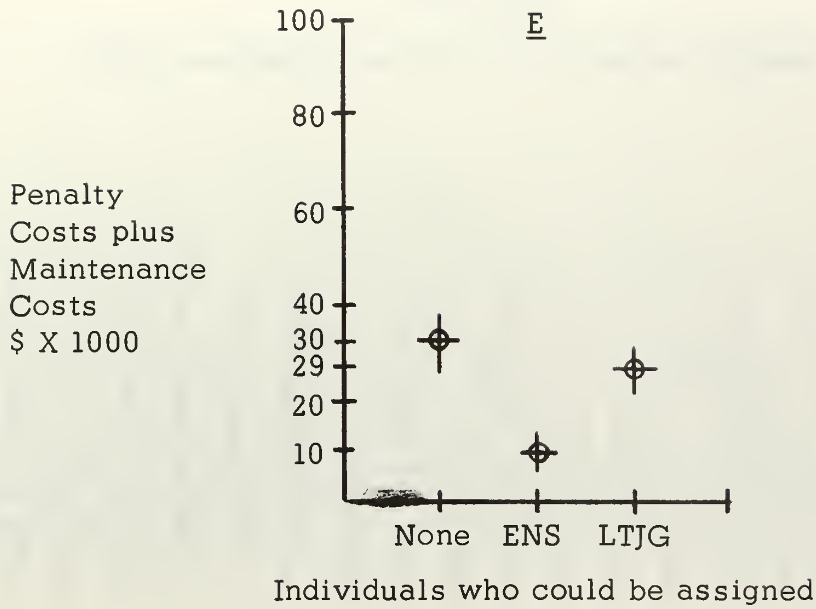






(3) Next construct a Figure 6 for each type of billet.


A
B
C
D
Penalty Costs plus Maintenance Costs $ X 1000
246
106
100
80
60
40
20
17
None
LCDR
CDR
153
108
84
60
40
15
None
LT
LCDR
CDR
Individuals who could be assigned
Penalty Costs plus Maintenance Costs $ X 1000
85
82
60
43
20
14
None
LTJG
LT
LCDR
60
49
45
27
20
12
None
ENS
LTJG
LT
Individuals who could be assigned



(4) Construct C (C_{ij}) as follows:

$$
\begin{array}{ll}
 & \text{Billets} \\
 & \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{array} \\
\text{Career States} & \begin{array}{l} \text{CDR} \\ \text{LCDR} \\ \text{LT} \\ \text{LTJG} \\ \text{ENS} \\ \text{NONE} \end{array}
\begin{pmatrix}
17 & 108 & & & \\
106 & 15 & 85 & & \\
 & 84 & 14 & 45 & \\
 & & 43 & 12 & 29 \\
 & & & 27 & 10 \\
246 & 153 & 82 & 49 & 30
\end{pmatrix}
\end{array}
$$

It would not be necessary to perform the graphical analysis above in order to construct (C_{ij}), but it was performed here to demonstrate the arguments of Section 3.1. Also, it should be noted that the blanks in (C_{ij}) could have been calculated, but inasmuch as our assumed assignment policy will preclude such arcs in the model, the missing costs were deliberately omitted.

APPENDIX C

EXAMPLE PROBLEM SOLUTION IN TABULAR FORM

ARC(ij)	V_i	$V_i - V_j$	C_{ij}	L_{ij}	f_{ij}	M_{ij}	STATE	INTERPRETATION
(1,2)	259	0	0	40	40	40	β	Set in y_5
(1,2)	259	0	0	0	32	∞	β	ENS recruit $t=1$
(1,3)	259	15	0	40	40	40	γ	Set in y_4
(1,3)	259	15	15	0	0	∞	β	LTJG Dir Input $t=1$
(1,4)	259	15	0	30	30	30	γ	Set in y_3
(1,4)	259	15	35	0	0	∞	α	LT Dir Input $t=1$
(1,5)	259	15	0	10	10	10	γ	Set in y_2
(1,5)	259	15	95	0	0	∞	α	LCDR Dir Input $t=1$
(1,6)	259	15	0	10	10	10	γ	Set in y_1
(1,6)	259	15	150	0	0	∞	α	CDR Dir Input $t=1$
(1,25)	259	0	0	0	69	∞	β	ENS recruit $t=2$
(1,26)	259	20	20	0	30	∞	β	LTJG Dir Input $t=2$
(1,27)	259	45	45	0	7	∞	β	LT Dir Input $t=2$
(1,28)	259	105	105	0	0	∞	β	LCDR Dir Input $t=2$
(1,29)	259	175	175	0	4	∞	β	CDR Dir Input $t=2$
(1,48)	259	0	0	0	84	∞	β	ENS recruit $t=2$
(1,49)	259	25	25	0	34	∞	β	LTJG Dir Input $t=3$
(1,50)	259	55	55	0	3	∞	β	LT Dir Input $t=3$
(1,51)	259	120	120	0	1	∞	β	LCDR Dir Input $t=3$
(1,52)	259	200	200	0	7	∞	β	CDR Dir Input $t=3$

ARC(ij)	V_i	$V_i - V_j$	C_{ij}	L_{ij}	f_{ij}	M_{ij}	STATE	INTERPRETATION
(1,71)	259	0	0	0	138	∞	β	ENS recruit t = 4
(1,72)	259	30	30	0	60	∞	β	LTJG Dir Input t = 4
(1,73)	259	65	65	0	12	∞	β	LT Dir Input t = 4
(1,74)	259	135	135	0	5	∞	β	LCDR Dir Input t = 4
(1,75)	259	225	225	0	11	∞	β	CDR Dir Input t = 4
(1,94)	259	0	0	0	96	∞	β	ENS recruit t = 5
(1,95)	259	0	35	0	0	∞	α	LTJG Dir Input t = 5
(1,96)	259	0	90	0	0	∞	α	LT Dir Input t = 5
(1,97)	259	0	165	0	0	∞	α	LCDR Dir Input t = 5
(1,98)	259	227	250	0	0	∞	α	CDR Dir Input t = 5
(2,117)	259	0	0	6	6	6	β	ENS losses t = 1
(2,7)	259	0	0	0	40	45	β	ENS to ENS t = 1
(2,3)	259	15	0	0	26	26	γ	ENS to LTJG t = 1
(3,8)	244	0	0	0	40	45	β	LTJG to LTJG t = 1
(3,117)	244	-15	0	20	20	20	α	LTJG losses t = 1
(3,4)	244	0	0	0	6	13	β	LTJG to LT t = 1
(4,117)	244	-15	0	2	2	2	α	LT losses t = 1
(4,9)	244	60	0	0	33	33	γ	LT to LT t = 1
(4,5)	244	0	0	0	1	6	β	LT to LCDR t = 1
(5,10)	244	95	0	0	8	8	γ	LCDR to LCDR t = 1

ARC(ij)	V_i	$V_i - V_j$	C_{ij}	L_{ij}	f_{ij}	M_{ij}	STATE	INTERPRETATION
(5,117)	244	-15	0	1	1	1	α	LCDR losses $t = 1$
(5,6)	244	0	0	0	2	3	β	LCDR to CDR $t = 1$
(6,117)	244	-15	0	3	3	3	α	CDR losses $t = 1$
(6,11)	244	143	0	0	9	9	γ	CDR to CDR $t = 1$
(7,12)	259	10	10	0	40	∞	β	ENS to ENS Billet $t = 1$
(7,13)	259	9	27	0	0	∞	α	ENS to LTJG Billet $t = 1$
(8,14)	244	24	29	0	0	∞	α	LTJG to ENS Billet $t = 1$
(8,15)	244	12	12	0	40	∞	β	LTJG to LTJG Billet $t = 1$
(8,16)	244	34	43	0	0	∞	α	LTJG to LT Billet $t = 1$
(9,17)	184	-16	45	0	0	∞	α	LT to LTJG Billet $t = 1$
(9,18)	184	14	14	0	30	∞	β	LT to LT Billet $t = 1$
(9,19)	184	84	84	0	3	∞	β	LT to LCDR Billet $t = 1$
(10,20)	149	49	85	0	0	∞	α	LCDR to LT Billet $t = 1$
(10,21)	149	15	15	0	7	∞	β	LCDR to LCDR Billet $t = 1$
(10,22)	149	106	106	0	1	∞	β	LCDR to CDR Billet $t = 1$
(11,23)	101	101	108	0	0	∞	α	CDR to LCDR Billet $t = 1$
(1,24)	101	17	17	0	9	∞	β	CDR to CDR Billet $t = 1$
(12,25)	249	-10	0	0	0	∞	α	None
(12,25)	249	-10	-30	0	40	40	γ	None
(13,25)	250	-9	-49	0	0	0	γ	None

ARC(ij)	V_i	$V_i - V_j$	C_{ij}	L_{ij}	f_{ij}	M_{ij}	STATE	INTERPRETATION
(14,26)	220	-19	-30	0	0	0	γ	None
(15,26	232	-7	0	0	0	∞	α	None
(15,26)	232	-7	-49	0	40	40	γ	None
(16,26)	210	-29	-82	0	0	0	γ	None
(17,27)	200	-14	-49	0	0	0	γ	None
(18,27)	170	-44	0	0	0	∞	α	None
(18,27)	170	-44	-82	0	30	30	γ	None
(19,27)	100	-114	-153	0	3	3	γ	None
(20,28)	100	-54	-82	0	0	0	γ	None
(21,28)	134	-20	0	0	0	∞	α	None
(21,28)	134	-20	-153	0	7	7	γ	None
(22,28)	43	-111	-246	0	1	1	γ	None
(23,29)	0	-84	-153	0	0	0	γ	None
(24,29)	84	0	0	0	0	∞	β	None
(24,29)	84	0	-246	0	9	9	γ	None
(25,117)	259	0	0	9	9	9	β	ENS losses t = 2
(25,30)	259	0	0	0	60	75	β	ENS to ENS t = 2
(25,26)	259	20	0	0	40	40	γ	ENS to LTJG t = 2
(26,31)	239	0	0	0	60	69	β	LTJG to LTJG t = 2
(26,117)	239	-20	0	30	30	30	β	LTJG losses t = 2

ARC(ij)	V_i	$V_i - V_j$	C_{ij}	L_{ij}	f_{ij}	M_{ij}	STATE	INTERPRETATION
(26,27)	239	25	0	0	20	20	γ	LTJG to LT t = 2
(27,32)	214	0	0	0	49	50	β	LT to LT t = 2
(27,117)	214	-45	0	2	2	2	α	Lt losses t = 2
(27,28)	214	60	0	0	9	9	γ	LT to LCDR t = 2
(28,33)	154	20	0	0	12	12	γ	LCDR to LCDR t = 2
(28,117)	154	-105	0	1	1	1	α	LCDR losses t = 2
(28,29)	154	70	0	0	4	4	γ	LCDR to CDR t = 2
(29,117)	84	-175	0	3	3	3	α	CDR losses t = 2
(29,34)	84	8	0	0	14	14	γ	CDR to CDR t = 2
(30,35)	259	10	10	0	60	∞	β	ENS to ENS Billet t = 2
(30,36)	259	9	27	0	0	∞	α	ENS to LTJG Billet t = 2
(31,37)	239	19	29	0	0	∞	α	LTJG to ENS Billet t = 2
(31,38)	239	12	12	0	60	∞	β	LTJG to LTJG Billet t = 2
(31,39)	239	19	43	0	0	∞	α	LTJG to LT Billet t = 2
(32,40)	214	34	45	0	0	∞	α	LT to LTJG Billet t = 2
(32,41)	214	14	14	0	45	∞	β	LT to LT Billet t = 2
(32,42)	214	84	84	0	4	∞	β	LT to LCDR Billet t = 2
(33,43)	134	54	85	0	0	∞	α	LCDR to LT Billet t = 2
(33,44)	134	15	15	0	11	∞	β	LCDR to LCDR Billet t = 2
(33,45)	134	106	106	0	1	∞	β	LCDR to CDR Billet t = 2

ARC(ij)	V_i	$V_i - V_j$	C_{ij}	L_{ij}	f_{ij}	M_{ij}	STATE	INTERPRETATION
(34,46)	76	76	108	0	0	∞	α	CDR to LCDR Billet t = 2
(34,47)	76	17	17	0	14	∞	β	CDR to CDR Billet t = 2
(35,48)	249	-10	0	0	0	∞	α	None
(35,48)	249	-10	-30	0	60	60	γ	None
(36,48)	250	-9	-49	0	0	0	γ	None
(37,49)	220	-14	-30	0	0	0	γ	None
(38,49)	227	-7	0	0	0	∞	α	None
(38,49)	227	-7	-49	0	60	60	γ	None
(39,49)	220	-14	-82	0	0	0	γ	None
(40,50)	180	-24	-49	0	0	0	γ	None
(41,50)	200	-4	0	0	0	∞	α	None
(41,50)	200	-4	-82	0	45	45	γ	None
(42,50)	130	-74	-153	0	4	4	γ	None
(43,51)	80	-59	-82	0	0	0	γ	None
(44,51)	119	-20	0	0	0	∞	α	None
(44,51)	119	-20	-153	0	11	11	γ	None
(45,51)	28	-111	-246	0	1	1	γ	None
(46,52)	0	-59	-153	0	0	0	γ	None
(47,52)	59	0	0	0	0	∞	β	None
(47,52)	59	0	-246	0	14	14	γ	None

ARC(ij)	V_i	$V_i - V_j$	C_{ij}	L_{ij}	f_{ij}	M_{ij}	STATE	INTERPRETATION
(48,117)	259	0	0	12	12	12	β	ENS losses t = 3
(48,53)	259	0	0	0	80	95	β	ENS to ENS t = 3
(48,49)	259	25	0	0	52	52	γ	ENS to LTJG t = 3
(49,54)	234	0	0	0	80	90	β	LTJG to LTJG t = 3
(49,117)	234	-25	0	40	40	40	α	LTJG losses t = 3
(49,50)	234	30	0	0	26	26	γ	LTJG to LT t = 3
(50,55)	204	0	0	0	62	66	β	LT to LT t = 3
(50,117)	204	-55	0	4	4	4	γ	LT losses t = 3
(50,51)	204	65	0	0	12	12	γ	LT to LCDR t = 3
(51,117)	139	-120	0	2	2	2	α	LCDR losses t = 3
(51,56)	139	20	0	0	18	18	γ	LCDR to LCDR t = 3
(51,52)	139	80	0	0	5	5	γ	LCDR to CDR t = 3
(52,117)	59	-200	0	6	6	6	α	CDR losses t = 3
(52,57)	59	8	0	0	20	20	γ	CDR to CDR t = 3
(53,58)	259	10	10	0	80	∞	β	ENS to ENS Billet t = 3
(53,59)	259	9	27	0	0	∞	α	ENS to LTJG Billet t = 3
(54,60)	234	14	29	0	0	∞	α	LTJG to ENS Billet t = 3
(54,61)	234	12	12	0	80	∞	β	LTJG to LTJG Billet t = 3
(54,62)	234	12	43	0	0	∞	α	LTJG to LT Billet t = 3
(55,63)	204	-18	45	0	0	∞	α	LT to LTJG Billet t = 3

ARC(ij)	V_i	$V_i - V_j$	C_{ij}	L_{ij}	f_{ij}	M_{ij}	STATE	INTERPRETATION
(55,64)	204	14	14	0	60	∞	β	LT to LT Billet t = 3
(55,65)	204	84	84	0	2	∞	β	LT to LCDR Billet t = 3
(56,66)	119	-47	85	0	0	∞	α	LCDR to LT Billet t = 3
(56,67)	119	15	15	0	18	∞	β	LCDR to LCDR Billet t = 3
(56,68)	119	77	106	0	0	∞	α	LCDR to CDR Billet t = 3
(57,69)	51	-53	108	0	0	∞	α	CDR to LCDR Billet t = 3
(57,70)	51	17	17	0	20	∞	β	CDR to CDR Billet t = 3
(58,71)	249	-10	0	0	0	∞	α	None
(58,71)	249	-10	-30	0	80	80	γ	None
(59,71)	250	-9	-49	0	0	0	γ	None
(60,72)	220	-9	-30	0	0	0	γ	None
(61,72)	222	-7	-49	0	80	80	γ	None
(61,72)	222	-7	0	0	0	∞	α	None
(62,72)	222	-7	-82	0	0	0	γ	None
(63,73)	222	28	-49	0	0	0	γ	None
(64,73)	190	-4	-82	0	60	60	γ	None
(64,73)	190	-4	0	0	0	∞	α	None
(65,73)	120	-74	-153	0	2	2	γ	None
(66,74)	166	42	-82	0	0	0	γ	None
(67,74)	104	-20	-153	0	18	18	γ	None

ARC(ij)	V_i	$V_i - V_j$	C_{ij}	L_{ij}	f_{ij}	M_{ij}	STATE	INTERPRETATION
(67,74)	104	-20	0	0	0	∞	α	None
(68,74)	42	-82	-246	0	0	0	γ	None
(69,75)	104	70	-153	0	0	0	γ	None
(70,75)	34	0	-246	0	20	20	γ	None
(70,75)	34	0	0	0	0	∞	β	None
(71,117)	259	0	0	18	18	18	β	ENS losses $t = 4$
(71,76)	259	0	0	0	120	135	β	ENS to ENS $t = 4$
(71,72)	259	30	0	0	80	80	γ	ENS to LTJG $t = 4$
(72,117)	229	-30	0	60	60	60	α	LTJG losses $t = 4$
(72,77)	229	0	0	0	120	132	β	LTJG to LTJG $t = 4$
(72,73)	229	35	0	0	40	40	γ	LTJG to LT $t = 4$
(73,117)	194	-65	0	6	6	6	α	LT losses $t = 4$
(73,78)	194	0	0	0	90	99	β	LT to LT $t = 4$
(73,74)	194	70	0	0	18	18	γ	LT to LCDR $t = 4$
(74,117)	124	-135	0	3	3	3	α	LCDR losses $t = 4$
(74,79)	124	3	0	0	30	30	γ	LCDR to LCDR $t = 4$
(74,75)	124	90	0	0	8	8	γ	LCDR to CDR $t = 4$
(75,117)	34	-225	0	9	9	9	α	CDR losses $t = 4$
(75,80)	34	0	0	0	30	33	β	CDR to CDR $t = 4$
(76,81)	259	10	10	0	120	∞	β	ENS in ENS Billet $t = 4$

ARC(ij)	V_i	$V_i - V_j$	C_{ij}	L_{ij}	f_{ij}	M_{ij}	STATE	INTERPRETATION
(76,82)	259	9	27	0	0	∞	α	ENS in LTJG Billet t = 4
(77,83)	229	-21	29	0	0	∞	α	LTJG in ENS Billet t = 4
(77,84)	229	12	12	0	120	∞	β	LTJG to LTJG Billet t = 4
(77,85)	229	29	43	0	0	∞	α	LTJG to LT Billet t = 4
(78,86)	194	-23	45	0	0	∞	α	LT to LTJG Billet t = 4
(78,87)	194	14	14	0	90	∞	β	LT to LT Billet t = 4
(78,88)	194	32	84	0	0	∞	α	LT to LCDR Billet t = 4
(79,89)	121	21	85	0	0	∞	α	LCDR to LT Billet t = 4
(79,90)	121	15	15	0	30	∞	β	LCDR to LCDR Billet t = 4
(79,91)	121	106	106	0	0	∞	β	LCDR to CDR Billet t = 4
(80,92)	34	24	108	0	0	∞	α	CDR to LCDR Billet t = 4
(80,93)	34	17	17	0	30	∞	β	CDR to CDR Billet t = 4
(81,94)	249	-10	-30	0	120	120	γ	None
(81,94)	249	-10	0	0	0	∞	α	None
(82,94)	250	-9	-49	0	0	0	γ	None
(83,95)	250	-9	-30	0	0	0	γ	None
(84,95)	217	-42	0	0	0	∞	α	None
(84,95)	217	-42	-49	0	120	120	γ	None
(85,95)	200	-59	-82	0	0	0	γ	None
(86,96)	217	-42	-49	0	0	0	γ	None

ARC(ij)	V_i	$V_i - V_j$	C_{ij}	L_{ij}	f_{ij}	M_{ij}	STATE	INTERPRETATION
(87,96)	180	-79	0	0	0	∞	α	None
(87,96)	180	-79	-82	0	90	90	γ	None
(88,96)	162	-97	-153	0	0	0	γ	None
(89,97)	100	-159	-82	0	0	0	α	None
(90,97)	106	-153	0	0	0	∞	α	None
(90,97)	106	-153	-153	0	30	30	β	None
(91,97)	15	-244	-246	0	0	0	γ	None
(92,98)	10	-22	-153	0	0	0	γ	None
(93,98)	17	-15	0	0	0	∞	α	None
(93,98)	17	-15	-246	0	30	30	γ	None
(94,117)	259	0	0	18	18	18	β	ENS losses $t = 5$
(94,99)	259	0	0	0	120	132	β	ENS to ENS $t = 5$
(94,95)	259	0	0	0	78	80	β	ENS to LTJG $t = 5$
(95,100)	259	0	0	0	120	132	β	LTJG to LTJG $t = 5$
(95,117)	259	0	0	60	60	60	β	LTJG losses $t = 5$
(95,96)	259	0	0	0	18	40	β	LTJG to LT $t = 5$
(96,101)	259	0	0	0	90	99	β	LT to LT $t = 5$
(96,117)	259	0	0	6	6	6	β	LT losses $t = 5$
(96,97)	259	0	0	0	12	18	β	LT to LCDR $t = 5$
(97,102)	259	0	0	0	30	33	β	LCDR to LCDR $t = 5$

ARC(ij)	V_i	$V_i - V_j$	C_{ij}	L_{ij}	f_{ij}	M_{ij}	STATE	INTERPRETATION
(97,117)	259	0	0	3	3	3	β	LCDR losses t = 5
(97,98)	259	227	0	0	9	9	γ	LCDR to CDR t = 5
(98,117)	32	-227	0	9	9	9	α	CDR losses t = 5
(98,103)	32	0	0	0	30	33	β	CDR to CDR t = 5
(99,104)	259	10	10	0	120	∞	β	ENS to ENS Billet t = 5
(99,105)	259	9	27	0	0	∞	α	ENS to LTJG Billet t = 5
(100,106)	259	10	29	0	0	∞	α	LTJG to ENS Billet t = 5
(100,107)	259	12	12	0	120	∞	β	LTJG to LTJG Billet t = 5
(100,108)	259	39	43	0	0	∞	α	LTJG to LT Billet t = 5
(101,109)	259	39	45	0	0	∞	α	LT to LTJG Billet t = 5
(101,110)	259	14	14	0	90	∞	β	LT to LT Billet t = 5
(101,111)	259	59	84	0	0	∞	α	LT to LCDR Billet t = 5
(102,112)	259	82	85	0	0	∞	α	LCDR to LT Billet t = 5
(102,113)	259	15	15	0	30	∞	β	LCDR to LCDR Billet t = 5
(102,114)	259	100	106	0	0	∞	α	LCDR to CDR Billet t = 5
(103,115)	32	-74	108	0	0	∞	α	CDR to LCDR Billet t = 5
(103,116)	32	17	17	0	30	∞	β	CDR to CDR Billet t = 5
(104,117)	249	-10	-30	0	120	120	γ	None
(105,117)	250	-9	-49	0	0	0	γ	None
(106,117)	249	-10	-30	0	0	0	γ	None

ARC(ij)	V_i	$V_i - V_j$	C_{ij}	L_{ij}	f_{ij}	M_{ij}	STATE	INTERPRETATION
(107,117)	247	-12	-49	0	120	120	γ	None
(108,117)	220	-39	-82	0	0	0	γ	None
(109,117)	220	-39	-49	0	0	0	γ	None
(110,117)	245	-14	-82	0	90	90	γ	None
(111,117)	200	-59	-153	0	0	0	γ	None
(112,117)	177	-82	-82	0	0	0	β	None
(113,117)	244	-15	-153	0	30	30	γ	None
(114,117)	159	-100	-246	0	0	0	γ	None
(115,117)	106	-153	-153	0	0	0	β	None
(116,117)	15	-244	-246	0	30	30	γ	None
(117,1)	259	0	0	0	723	∞	β	None

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13. ABSTRACT

This thesis demonstrates the feasibility of constructing a network flow model to represent the U.S. Navy officer personnel system. This model consists of nodes connected by directed arcs which represent, respectively, career states and paths between these states. Flows moving over these arcs represent the movements of officers from state to state through time. A measure is developed which relates planning effectiveness to the dollar costs incurred by the Navy in recruiting, training, and maintaining officers. The network flow model is then equated to a linear program which can be solved for the dynamic flows of officers necessary to meet expected future requirements with maximum planning effectiveness. An example problem is hypothesized and solved to illustrate the technique. The author recommends that a small scale operational model be constructed to represent a segment of the Navy Officer Corps in order to better estimate the value of this approach to officer personnel planning in the Navy.

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